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CHARACTERISTICS OF FLOW OVER THE CREST OF A SIPHON SPILLWAY

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Abstract: Siphon spillways are highly efficient in controlling water flow with small increases in the reservoir water level. Flow over the circular crest is highly curvilinear. The purpose of this paper is to study the characteristics of curvilinear flow over a siphon in the region of round-crest. To investigate the curvilinear flow over the crest, theoretical analysis and numerical modelling were performed. The theoretical analysis was conducted to determine the configuration of streamlines and the radii of curvature of the streamlines in curvilinear coordinates. The flow field was obtained using Computational Fluid Dynamics (CFD) model for the case of submerged flow over a range of Reynolds number values. The computations use the RNG k-ε model for turbulence closure. The RNG k-ε model is proven to have high accuracy for simulations of turbulent flow, especially in domains with curved boundaries and wall boundary layers. The CFD model predictions are validated by using results obtained from theoretical analysis of the siphon model. The results from the current study shed light on such characteristics of the flow past the siphon crest as the discharge coefficient, velocity distributions and pressure distributions. It is shown that the pressure on the siphon crest is negative and therefore could lead to the occurrence of cavitation when it reaches the vapour pressure of water. The results show that the flow field based on the RNG k-ε model matches well with the data from the theoretical analysis.

1 INTRODUCTION

Siphon spillways are a type of spillway that is critical to the safety of the reservoir behind a dam. They function to discharge an excessive amount of water that cannot be passed through a diversion channel or be stored in the reservoir. A siphon spillway has highly curved boundaries. As water passes through the siphon conduit, the flow is turbulent, the velocity structures are non-uniform, and the pressure field deviates significantly from a hydrostatic distribution. To deal with such complex flows, one may use three different but complementary techniques: mathematical analysis, laboratory measurements, and field observation. The purpose of this paper is to study the characteristics of curvilinear flow over a siphon spillway in the regain of a rounded-crest. We take the mathematical analysis approach.

Regarding complex curvilinear flows, previous researchers have conducted some analytical and numerical studies. (Cassidy 1965) reported an analytic solution to ideal fluid flow over a circular crested spillway. The purpose of the solution was to determine the discharge coefficients, free-surface profiles and pressure distributions resulting from irrotational flow. (Cassidy 1970) presented a mathematical method for the analysis of the flow over a standard-shaped spillway. Currently, our understanding is limited with respect to the hydraulic characteristics of the spillway crest, especially at water head higher than the design head to control pressure to remain less than atmospheric pressure.

Dressier (1978) derived equations for flow over curved open channel surface. The equations include new additional terms to express the curvature and its derivative of the channel floor. In the new equations, (Dressier 1978) represented the pressure term to include the effect of the streamline's curvature in addition to the hydrostatic pressure. A theoretical model was developed to determine the water surface profiles and the radius of curvature of the streamlines in curvilinear coordinates. (Dressler's 1978) equation was used to study the flow at the crest regain mathematically. The equation is significant in flow analysis and solving many hydraulic problems related to the curved surfaces. In existing theoretical models, the momentum analysis of flow, velocity field in region of the crest and provided pressure distribution were devoted to study flow over various types of free flow structures such as broad crested weir and long crested weir. Yet, it is very important to study the curvilinear flow behaver over the crest in case of the siphon spillway. (Savage and Johnson 2001) have studied the flow behavior over crest of spillway using a scaled physical model and a two-dimensional numerical model.

In this study, he compared the results of analytical and numerical models with existing literature results for flow over an uncontrolled ogee crest. Performance data were interpolated from reports published by the U.S. Army Corps of Engineers (USACE 1990), and U.S. Bureau of Reclamation (USBR 1977). More recently, a few numerical studies have also been reported. For instance, (Tadayon and Ramamurthy 2012) used the RNG k-ε model to obtain the coefficient of discharge for a siphon spillway. They validated their model predictions using results of experiments related to a physical model of the siphon spillway. They used a physical siphon model and 3D numerical model of siphon spillway and investigated curvilinear flow and the discharge coefficient. The data obtained from the test on a laboratory siphon model was used to evaluate the results predicted by the numerical simulations. In existing theoretical and numerical models, curvilinear flow is well addressed for curved boundaries for different hydraulic structures such as weirs and free overflow spillway. Yet, it is very important to study the curvilinear flow behaver over the crest in case of the siphon spillway.

2 THEORETICAL CURVILINEAR FLOW MODEL

In open channels and many hydraulic structures, the streamlines are straight and parallel to the channel bed. The pressure at any point on the cross section of the flow can be determined as water depth from this point to the free surface using pressure piezometers. In this case the pressure distribution is liner and presented as straight line. Conversely, in the region of siphon's crest, the streamlines have obvious curvature and the flow is known as curvilinear flow. In such flows the pressure distribution over the crest deviates from liner hydrostatic pressure distribution to nonlinear pressure distribution. Therefore, for flow over the crest the functional representation of the flow streamlines can be expressed as,

[1] $y = f(x)$

where y and x are the vertical and horizontal axis at the crest.

For any streamline passing through any point (x, y) the radius of curvature of the streamline r and the inclination ∅ can be determined as,

[2] $tan\,\emptyset = f'(x)$

[3] $r = \frac{\left[1+\left(f'(x)\right)^2\right]^{3/2}}{f''(x)}$

where $f'$and f'' are first and second derivatives of streamlines function.

2.1 Velocity distribution

To compute the velocity over the crest, following theoretical assumptions have been made:

(1) Flow is steady and two-dimensional flow;

(2) Water viscosity, water surface tension, and solid surface roughness are negligible; and

(3) The crest boundary layer thickness δ is very small compared to water depth at the crest Y_2 where Y_2 equals the siphon throat height D.

The Dressler equation was used to mathematically study the flow at the crest regain. Following the steps indicated by (Dressier 1978) and (Ramamurthy (1993) the flow rate can be written as

[4] $q = U\, R\, ln\left(1 + \frac{D}{R}\right)$

where U is the maximum velocity above the crest, R is the crest radius and D is the water depth above the crest. Equation 5 shows the maximum velocity of the flow passing the siphon crest.

[5] $U = \sqrt{2g\,\left(H_1 - \left({}^{P}/_{\gamma}\right)_{CR}\right)}$

The horizontal velocity u at any depth y is expressed as:

[6] $u = \frac{U}{(1+\frac{y}{r})}$

where U is the maximum velocity above the crest, u is the horizontal velocity at the depth y, and R is the crest radius (Dressler 1978). Horizontal velocity u can be normalized by U_{max}. Hence, velocity profile can be written as:

[7] $\frac{u}{U} = \frac{[1-\left(\frac{p}{\gamma H_1}\right)_{cr}]^{0.5}}{(1+\frac{y}{R})}$

2.2 Pressure distribution

At the siphon crest, the flow is concave flow where the centrifugal forces are reinforcing the gravity action. As a result, the pressure at the crest section is greater than hydrostatic pressure and it can be computed as:

[8] $P = \frac{wd}{g}\frac{u^2}{r}$

Therefore, if P_1 and P_2 are the pressures at two points on the vertical centerline of the crest, the pressure-gradient is $(P_1$-$P_2)/r$, where u is the horizontal velocity and r is the radial distance. we consider the pressure gradient as

[9] $\frac{dp}{dr} = \frac{u^2}{r}$

and the total head is

[10] $H = \left[\frac{1}{\gamma}\left(p_{cr} + \frac{dp}{dr}\right)\right] + \frac{u^2}{2g}$

2.3 Hydraulic conditions

Data from an experimental investigation for exact siphon spillway geometry are used as initial input for the numerical model. The direct water depth measurements at downstream and upstream and the water discharge are shown in the Table1.

Table 1: siphon spillway hydraulic conditions

Flow rate (m^3/s)	Upstream water level (m)	Downstream water level (m)
0.024	0.565	0.203
0.025	0.557	0.228
0.024	0.518	0.156
0.022	0.569	0.260
0.024	0.612	0.275
0.023	0.267	0.167
0.025	0.356	0.222

3 NUMERICAL MODEL

3.1 Numerical setup

A three-dimensional geometry model was reproduced and analyzed. The powerful available computational fluid dynamics (CFD) software FLUENT-18.1 closes the system and approximate the RANS to equations that can be solved numerically. The power law function that generates finer mesh close to the solid boundaries was used to mesh the model domain. Unstructured mesh is used in different sizing for the mesh development and a fine mesh is used at the zone near to the internal surface of the siphon crest and crown. The pressure–velocity conjugation is carried out using the pressure-implicit with splitting of operators (PISO) algorithm (Ferziger 2002). In addition, the shape and location of the free surface at downstream section and upstream section, where the boundaries are two phases (air and water), were computed as part of the solution using volume-of fluid (VOF) method. This method rests on conceptualizations involving a fractional volume of fluid. Velocity and pressure data for a siphon spillway were obtained using RNG k-ε model where RNG refers to Re-Normalisation Group methods that used to renormalize the Navier-Stokes equations. The model predictions were validated using the analytical results.

3.2 Boundary conditions

For flow simulations, it is important that the boundary conditions accurately represent the real prototype and its physical conditions. The conditions at the boundaries (Figure 1) in the numerical model are as follows: a) upstream of the siphon, the left boundary is modeled as a velocity inlet with pressure based on water depth upstream of the siphon; b) at the outlet, downstream of the siphon the boundary is pressure outlet; c) sidewalls and the bottom of the channel are no-slip walls; d) similarly, the boundary condition of the siphon body is a no-slip wall. Siphon walls modeled as no slip are defined as zero tangential and normal velocities ($u = v = w = 0$). With a no-slip boundary, it is assumed that the law of the wall profile exists in the boundary region. Since the purpose of these simulations is to model flow over the siphon with deferent water levels, all the air boundaries are defined as pressure boundaries with gauge pressure equal to zero.

To numerically solve the two-phases flow, it is important that the interphase between the two fluids be accurately determined. In case of location, and the boundary conditions. One mean method to compute the free surface is the Volume of Fluid (VOF) method. The VOF method defines cells of the domain depending on the volume of one the fluids in the cell. The cells can be empty with value of zero or full cell with value of 1. Cells are assigned as partially filled cells with a value between zero and 1 [5]. Upstream and downstream flows are two phases (air and water) and the interphase between the two fluids fluctuates with time. For the initial condition, the free surface location is known (the pressure distribution is known at the inlet).

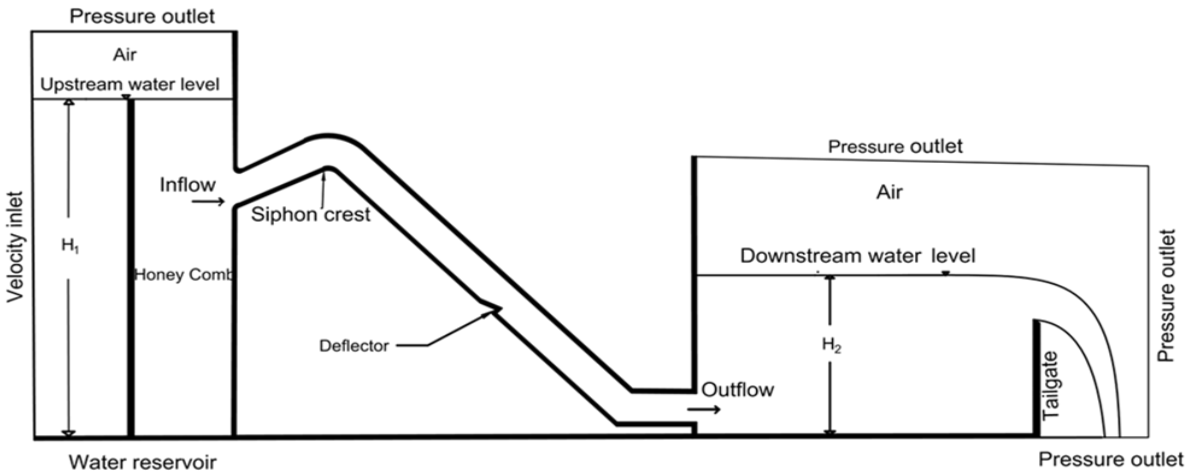

Figure 1: Longitudinal section of the siphon spillway

4 RESULTS

The horizontal velocity component u was computed using Computational Fluid Dynamics (CFD) techniques at the centerline of the siphon's crest C and two locations on both sides (upstream and downstream) of the centerline at a distance of 0.01 m and 0.02 m, respectively, as shown in Figure 2.

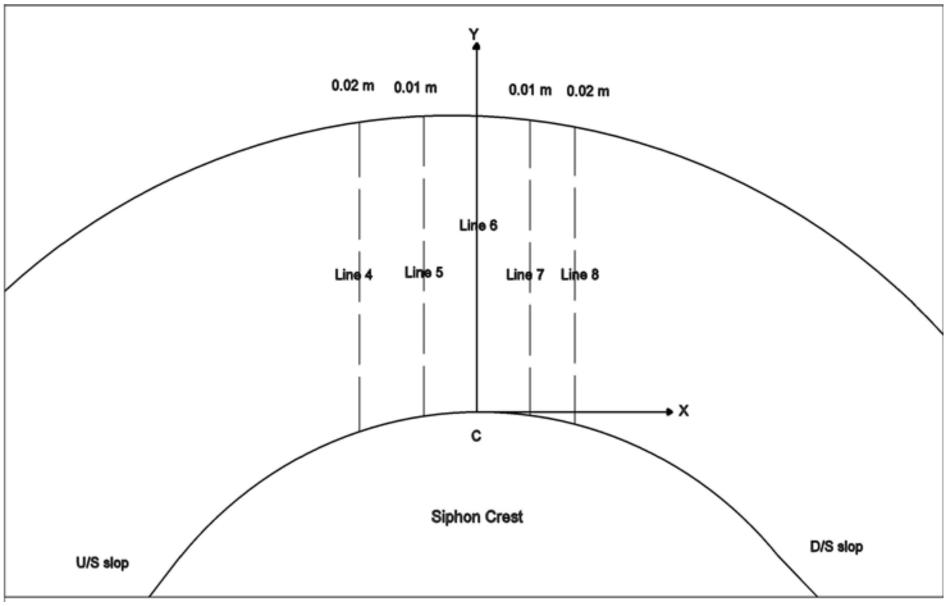

Figure 2: Locations of velocity data along the siphon crest

The numerical results of velocity profiles for the siphon model in the range of $0.5 \leq H1/D \leq 6.3$ are shown in Figure 3. The horizontal velocity u and water depth are normalized by U_{max} and the siphon throat depth D, respectively. The velocity profiles show a rapid increase in the area near the crest where is the maximum velocity was obtained at the very small height δ. In the zone above the boundary layer thickness δ, the velocity decreases gradually before it starts to rapidly decrease at, approximately, 99% of the total water depth.

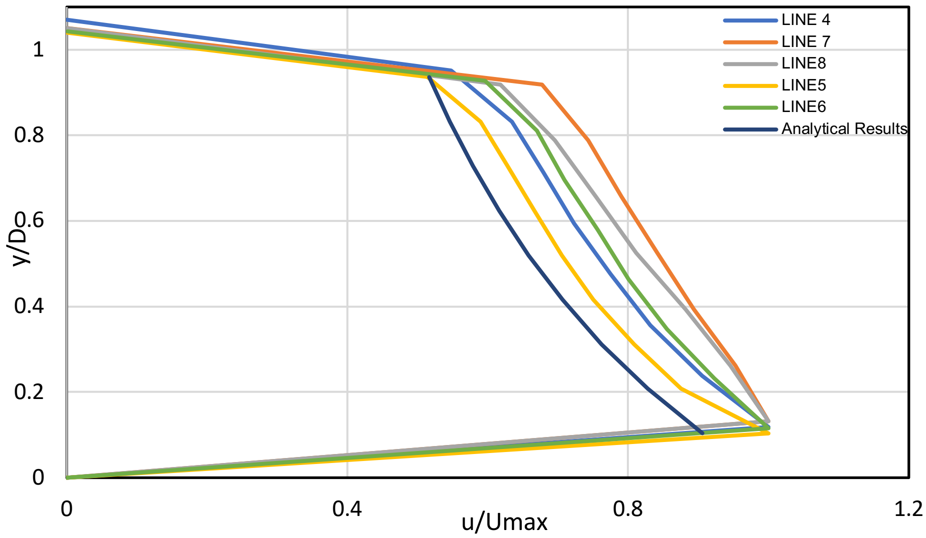

Figure 3: Analytical and numerical results of variation of y/D with u/U_{max} at the crest of the siphon spillway

Furthermore, the Dressler equation was used to determine the flow characteristics over the siphon crest such as the velocity distribution and to validate the numerical model predictions. Using the results that were obtained from Dressler equation as the observed standard, the percentages of error in maximum velocity values were determined. Table 2 shows the comparison of the predicted model results with the analytical results for the maximum velocity values close to the crest surface for several runs and different locations along the crest of the siphon. The relative percent error is defined as $(U_c - U_{th}) / U_{th} * 100$, where U_c is the maximum velocity in the numerical model and U_{th} is the analytical maximum velocity. The analytical maximum velocity is calculated using the Eq. [5]. According to this comparison, there is a reasonable agreement between the theoretical data and the numerical results.

Table 2: comparison of the predicted model results with the calculated results for the maximum velocity values

line	Calculated U_{max} (m/s)	Predicted U_{max} (m/s)	Relative error %
8	3.29	2.79	-15%
7	3.69	3.04	-17%
4	3.74	3.23	-13%
5	3.59	2.79	-22%
6	3.46	2.99	-13%

The results show that, because the flow is curvilinear flow in the region of the siphon crest, the pressure is non-hydrostatic. Therefore, pressure data were obtained from the numerical results of the velocity distribution. Figure 4 shows the variation of the pressure at several locations above the crest. In Figure 4, the x-axis indicates the pressure head P/yd and the y-axis is the water depth to the total head at the siphon crest. The pressure near the crest surface is negative pressure with value of -16.57 mmHg approximately. Minimum pressure values where the velocity is maximum are recorded near the crest. The negative pressure close to the crest can decrease to reach the vapor pressure of water where the cavitation is expected.

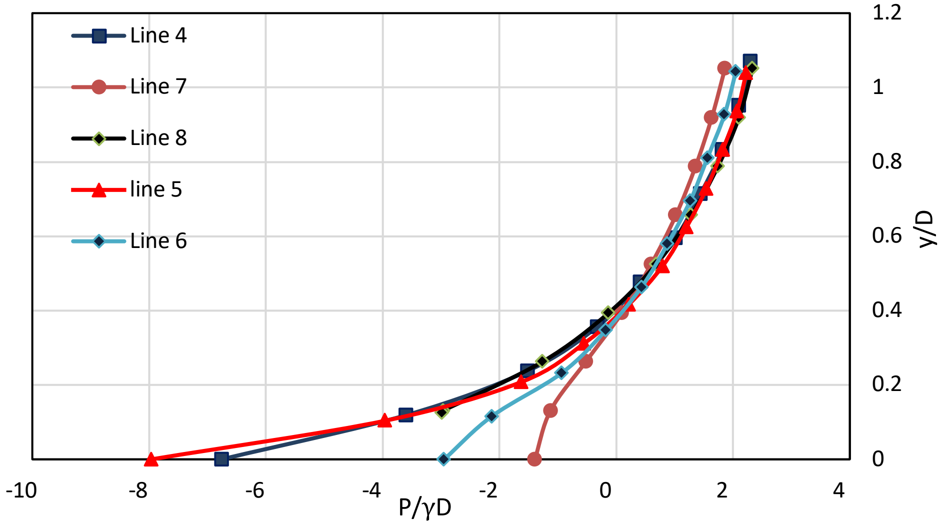

Figure 4: Pressure distribution at the crest section

Figure 5 shows a comparison between the computational and theoretical results of the pressure in the region of the siphon crest. In this comparison, the computational result gave approximated results of the calculated theoretical result.

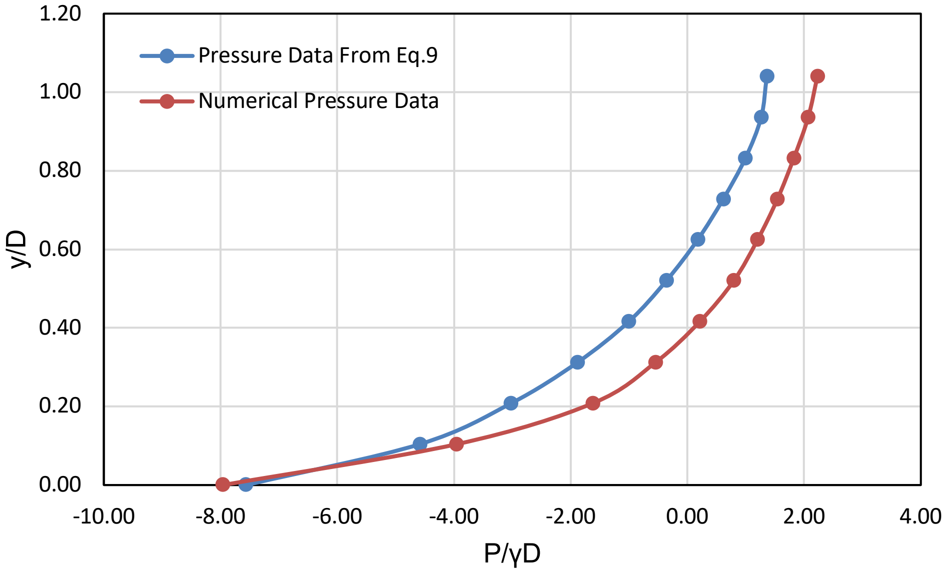

Figure 5: Analytical and numerical results of pressure distribution at the crest section

5 CONCLUSION

From numerical simulations of flow passing the siphon spillway, velocity profiles, maximum velocity values and pressure distribution were obtained. The velocity data and wall pressure data, the pressure distribution in the crest region were indirectly estimated and the minimum pressure value was obtained. The theoretical results to determine maximum velocity values U_{max}, streamlines' curvature and pressure distribution were determined using the Dressler equation. The analytical results were used to validate the numerical results and a reasonable agreement has been achieved.

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