Building Tomorrow's Society
Bâtir la Société de Demain

Fredericton, Canada
June 13 – June 16, 2018/ *Juin 13 – Juin 16, 2018*

THEORETICAL STUDY OF HYDRAULIC JUMPS IN STEEP STORM SEWERS

Wang, Chun Li[1,2], Li, S. Samuel[3]
[1] Graduate Student, Dept. of Building, Civil and Environmental Engineering Concordia Univ., 1455 de Maisonneuve Blvd. W., Montreal, QC, Canada H3G 1M8
[3] Professor, Dept. of Building, Civil and Environmental Engineering Concordia Univ., 1455 de Maisonneuve Blvd. W., Montreal, QC, Canada H3G 1M8
[2] spring3897@gmail.com

Abstract: In the design of storm sewers, pipes are often laid on slopes dictated by the actual slope of the terrain, and the slopes can be very large in some urban drainage systems of mountainous areas. The flow in a steep sewer pipe can be easily disturbed, causing an abrupt change from supercritical to subcritical state through a hydraulic jump. Downstream of the hydraulic jump, the flow can come into contact with the pipe vertex, and thus flow choking occurs. This paper focuses on a theoretical analysis of the above-mentioned flow phenomenon involved two connected sewer pipes laid on different slopes. For given values of pipe diameter, discharge, and slope, pertinent variables of flows in the pipes are determined by using the Manning's formula and the momentum equation. Values of the Froude number, F_r, in response to an increase in the slope of the upstream pipe and/or in discharge from upstream, are calculated. The occurrences of hydraulic jumps as well as their downstream sequent depths are predicted. The maximum filling ratios of upstream flow for which choking occurs at a series of slopes are determined. This paper contributes to practical solutions to the increasingly common problem of waterlogging in urban centres.

1 INTRODUCTION

The Froude number, F_r, is an important parameter in the analysis of free surface flow. The flow is subcritical if $F_r < 1$, critical if $F_r = 1$, and supercritical if $F_r > 1$. The transition from a subcritical to supercritical state is smooth and continuous. Inversely, the change from a supercritical to subcritical state is abrupt and discontinuous through a hydraulic jump (Chow, 1959). In a hilly region, the designs of storm sewer systems are often subject to the constraint of the actual slopes of the terrain. A very steep terrain slope tends to cause supercritical flows in sewer pipes laid on the terrain.

The poor hydraulic performance of supercritical flows in sewer pipes laid on steep slopes is often neglected, which can be easily disturbed by flow perturbations. The perturbations can result from such conditions as changes in flow direction and bottom slope, sewer junctions, or sewer sedimentation (Hager, 1999b). These perturbations may cause hydraulic jumps. If the sequent depth of the jump reaches the sewer soffit, flow choking occurs, and results in a sudden and abrupt transition from free surface flow to pressurized flow.

For given constant discharge and pipe diameter, the steeper the slope, the larger the value of F_r, which will have a higher risk of choking. The upper bound of filling ratio in many design guidelines of storm sewers is approximately 85% (ASCE, 1982). This is based on the concept of uniform flow, without considering the complex flow patterns of supercritical flow. The focus of this paper is on choking phenomenon caused by hydraulic jumps in steep storm sewers.

In the following, calculations are carried out for circular concrete sewers, with an internal diameter of $d_0 = 0.6$ m, a Manning's roughness factor of $n = 0.013$. The results are relevant flow variables in response to an increase in the slope of the upstream pipe and/or in discharge from upstream. The results show the condition under which flow choking occurs.

2 METHODS

2.1 Calculations of the Froude Number

For circular pipes, the geometric elements are defined in Figure1:

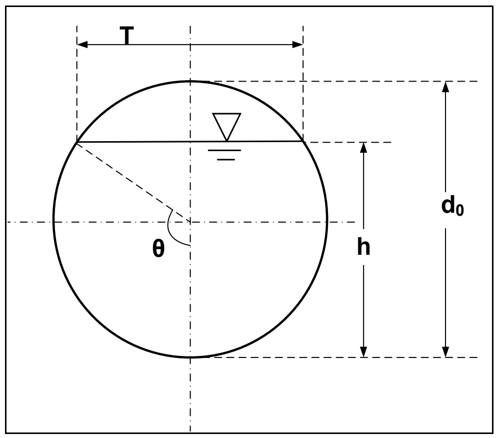

Figure1. Geometric elements of a circular section

The angle θ is related to the depth of flow, h, and diameter d_o as

[1] $$\theta = \pi - \cos^{-1}\left[\left(h-\frac{d_0}{2}\right)/\left(\frac{d_0}{2}\right)\right] = \pi - \cos^{-1}(2y-1)$$

where y is the filling ratio, defined as $y = h/d_0$. The flow area, A, is given by

[2] $$A=\frac{1}{8}(2\theta-\sin 2\theta)d_0^2$$

The hydraulic depth is related to the angle and pipe diameter as

[3] $$D=\frac{1}{8}\left(\frac{2\theta-\sin 2\theta}{\sin\theta}\right)d_0$$

The hydraulic radius varies with the angle and pipe diameter as

[4] $$R=\frac{1}{4}\left(1-\frac{\sin 2\theta}{2\theta}\right)d_0$$

Assume that the sewer pipe is long enough to develop normal flow from upstream. The Manning's formula can be used to calculate discharge Q:

[5] $$Q=\frac{1}{n}AR^{2/3}S_o^{1/2}$$

where S_0 in the bottom slope.

By definition, the Froude number F_r is

[6] $Fr = \frac{Q}{\sqrt{g(A^3/T)}}$

where V is the cross-sectionally averaged flow velocity, and g is the gravitation acceleration (equals to 9.81 m/s^2).

Expressions for Q and F_r in terms of θ can be derived by substituting Equations [2], [3], [4] into Equations [5] and [6]:

[7] $Q = \frac{1}{n}[(d_0^2/8)(2\theta\text{-}\sin 2\theta)]^{5/3}(\theta d_0)^{-2/3}S_0^{1/2}$

[8] $Fr = \frac{1}{n}\ g^{-1/2}S_0^{1/2}(sin\theta)^{1/2}(2\theta)^{-2/3}[2d_0(2\theta\text{-}sin2\theta)]^{1/6}$

As n, g, and d_0 are constant variables, the Froude number F_r only varies with θ and S_0. However, Expression [8] is too complicated for practical applications. A simplified expression for F_r is introduced

[9] $Fr = Q/(gd_0h^4)^{1/2}$

which permits the determination of Fr from discharge Q, flow depth h, and pipe diameter d_0 (Hager,1999b).

Depending on the approach Froude number, hydraulic jumps of different characteristics are possible to occur in sewer pipes. For $1 \le Fr \le 1.5$, the jump is an undular jump. For $1.5 \le Fr \le 2$, the undulations break at the upstream side. For $Fr > 2$, the undulations disappear and direct hydraulic jumps occur (Hager, 1999a).

2.2 Undular hydraulic jump

Undular hydraulic jumps feature successive shockwaves, which are highly sensitive to small perturbations, and hence undesirable. Such jumps may occur in a steep sewer. As illustrated in Figure 2, the main parameters of the jumps are wave crests h_{1C}, h_{2C}, and h_{3C}, and wave troughs h_{1t}, h_{2t}, and h_{3t} along the streamwise direction. Because of a wave diffusion process, the depths of wave peaks decrease in the flow direction (Hager,1999b). The elevation of the first wave crest is the highest and has the potential to cause flow choking. According to Gargano and Hager (2002), the first wave peak is related to the Froude number Fr and filling ratio y of the upstream flow as

[10] $h_{1c} = 1.20\, Fr_1\, y_1 \text{-}\ 0.10$

where the subscript 1 in the Froude number and filling ratio refer to the upstream flow.

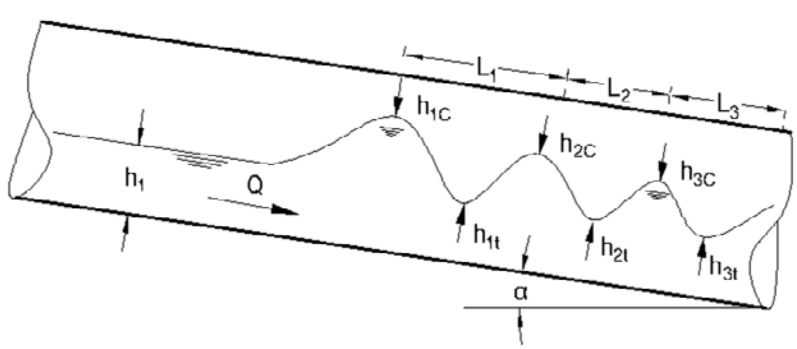

Figure 2. Characteristics of undular hydraulic jump in a sloping sewer, showing the elevations of wave crests h_{1c}, h_{2c}, and h_{3c}, wave troughs h_{1t}, h_{2t}, and h_{3t} above the invert of the sewer pipe, and distances L_1, L_2, and L_3 between adjacent wave crests. Here, h_1 is the depth of the upstream flow; α is the angle between the horizontal and the bottom of the sewer.

2.3 Direct hydraulic jump

In Figure 3, a direct hydraulic jump is illustrated. The flow is supercritical at depth h_{j1} just before the jump, and is subcritical at depth h_{j2} just after the jump. The jump has a length of L_j.

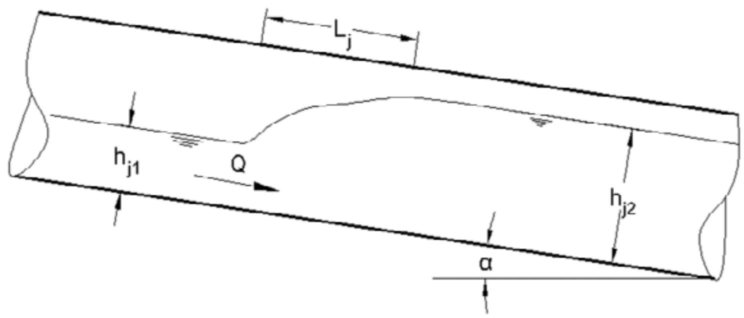

Figure 3. Direct hydraulic jump in a sloping sewer.

Whether or not the downstream flow is choked depends on h_{j2}, in comparison to the pipe diameter. The condition is similar to the first wave crest of an undular hydraulic jump. Between the upstream cross section (denoted by the subscript 1) and downstream cross section (denoted by the subscript 2), energy loss is unknown a prior, and thus the energy equation cannot be used straightforwardly to solve the hydraulic jump problem. However, the momentum equation can be used to calculate the sequent depth h_{j2}. The momentum equation is:

[11] $$QV_1 + \rho g Y_{c1} A_1 cos\alpha = QV_2 + \rho g Y_{c2} A_2 cos\alpha + L_j \frac{A_1+A_2}{2} \rho g\ sin\alpha - F_f - F_e$$

where ρ is the density of water; Y_c is the distance from the water surface to the centroid of the cross section; $sin\alpha = S_o$; F_f is friction force; F_e is any other external force. Compared to the momentum flux and pressure forces, F_f can be neglected because L_j is typically small. It is assumed that no other external forces are involved.

For the purpose of evaluating the gravity term in Equation [11], it is acceptable to assume that $A_1 = A_2 = \pi d_0^2/4$. With this approximation, Equation [11] can be rewritten as:

[12] $$\frac{Q^2}{gA_1} + Y_{c1}A_1\sqrt{1-S_0^2} = \frac{Q^2}{gA_2} + Y_{c2}A_2\sqrt{1-S_0^2} + \frac{1}{4}\pi d_0^2 L_j S_o$$

The term Y_cA can be determined as:

[13] $$Y_cA=\frac{d_0^3}{24}(3sin\theta-sin^3\theta-3\theta cos\theta)$$

Hager (1999b) suggested an empirical expression of the jump length L_j:

[14] $$L_j = 1.9h_1[2e^{1.5y_1} + e^{-10S_o}-1]\,(Fr_1 - 1)$$

The value of θ_1 can be determined for a given discharge Q. Because the parameters Fr, h, A are only functions of θ, F_{r1}, h_1, A_1 are known for the upstream flow. Thus, Equation [12] has only one unknown variable θ_2, but it cannot be solved explicitly. In this paper, Matlab code was developed and used to find solutions.

3 RESULTS

3.1 The Accuracy of Calculated Froude Number

In Figure 4, the values of *Fr* based on Equations [7] and [9] are compared. For the usual range of 30% < *y* < 80%, the relative errors of the approximate *Fr* values from Equation [9] are less than ±4%. This is acceptable.

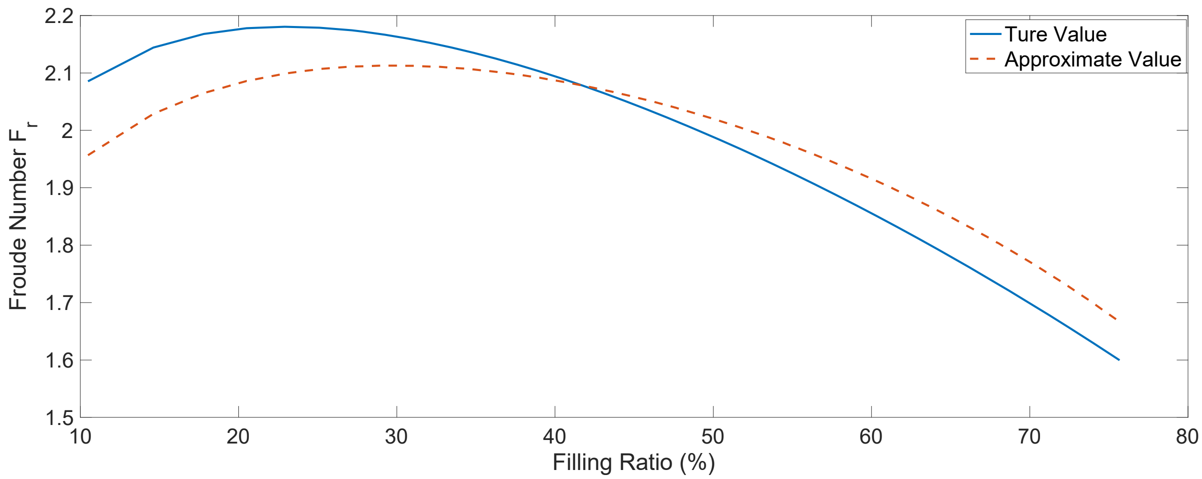

Figure 4. Froude number versus filling ratio for $d_0 = 0.6$ m, $S_0 = 2\%$, and $n = 0.013$.

3.2 The Effect of Bottom Slope on the Froude Number

For a constant value of bottom slope S_o, values for the Froude number F_r can be determined, by using [9], for a series values of discharge *Q*. The results of F_r versus *Q* are shown in Figure 5. For a constant bottom slope S_0, the Froude number F_r increases with increasing discharge *Q* until a certain value *Q*, and then decreases with increasing Q. As expected, for the same discharge Q, the Froude number F_r increases with increasing slope S_o.

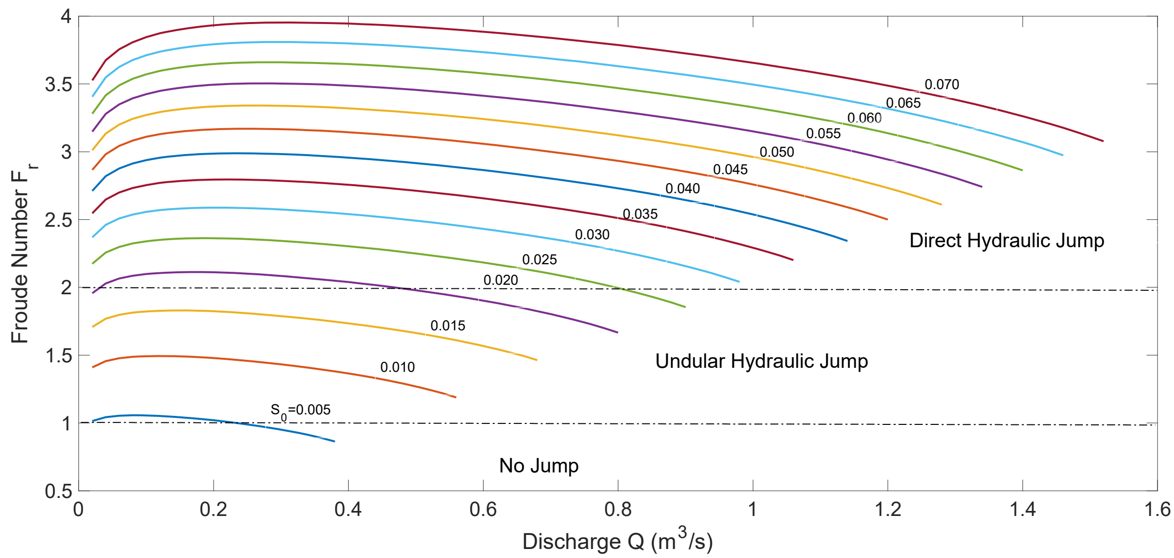

Figure 5. Froude number versus discharge Q from bottom slope S_0=*0.005* to S_0=*0.070*.

Note that the Froude number is between 1 and 1.5 for undular jumps, between 1.5 and 2 when the undulations break at the upstream side, and larger than 2 when the undulations disappear and direct hydraulic jumps occur. According to these criteria, flows in storm sewer pipes laid on slopes in the range of S_0 = 0.005 to 0.070 are divided into three categories: 1) For S_0 < 0.005, no hydraulic jump occurs; 2) for 0.005 ≤ S_0 ≤ 0.017, undular hydraulic jumps are possible to occur; 3) for 0.017 ≤ S_0 ≤ 0.07, direct hydraulic jumps are possible.

3.3 Downstream filling ratio

The first crest depth h_{1c} for an undular hydraulic jump and the sequent depth h_{j2} for a direct hydraulic jump for varying bottom slope can be calculated by using Matlab. And the relation between the downstream filling ratio y_2 and upstream filling ratio y_1 are graphically shown in Figure 6 and Figure 7. Similar results have been reported earlier in the literatures (Hager,1999b; Lowe,2011).

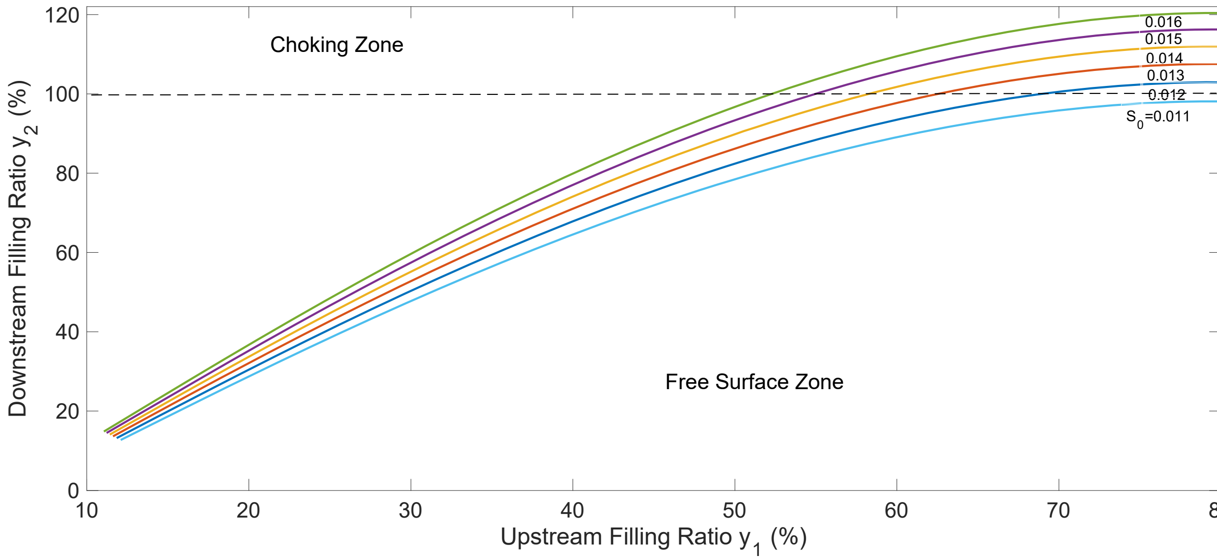

Figure 6. Downstream filling ratio for an undular hydraulic jump versus upstream filling ratio from bottom slope S_0=*0.011* to S_0=*0.016*.

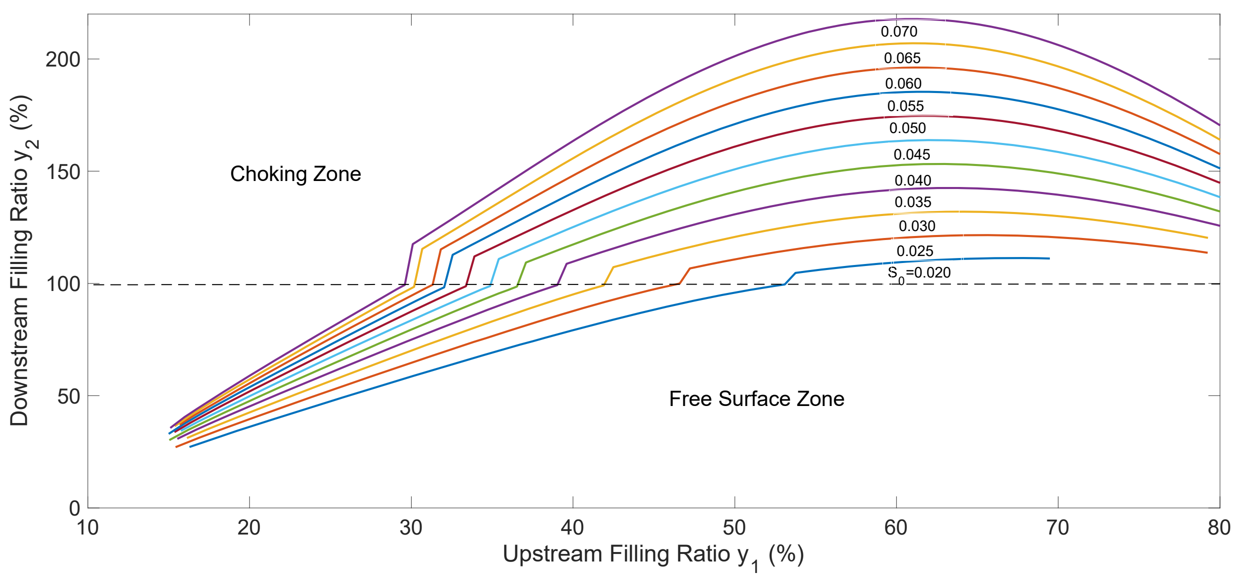

Figure 7. Downstream filling ratio for a direct hydraulic jump versus upstream filling ratio from bottom slope S_0=*0.020* to S_0=*0.070*.

3.4 Downstream flow choking

A sewer chokes whenever $h_{1C} \geq d_0$ or $h_{j2} \geq d_0$. Based on Equations [10] and [12], the corresponding filling ratio for choking y_c can be calculated in correspondence to an increase in bottom slope S_o.

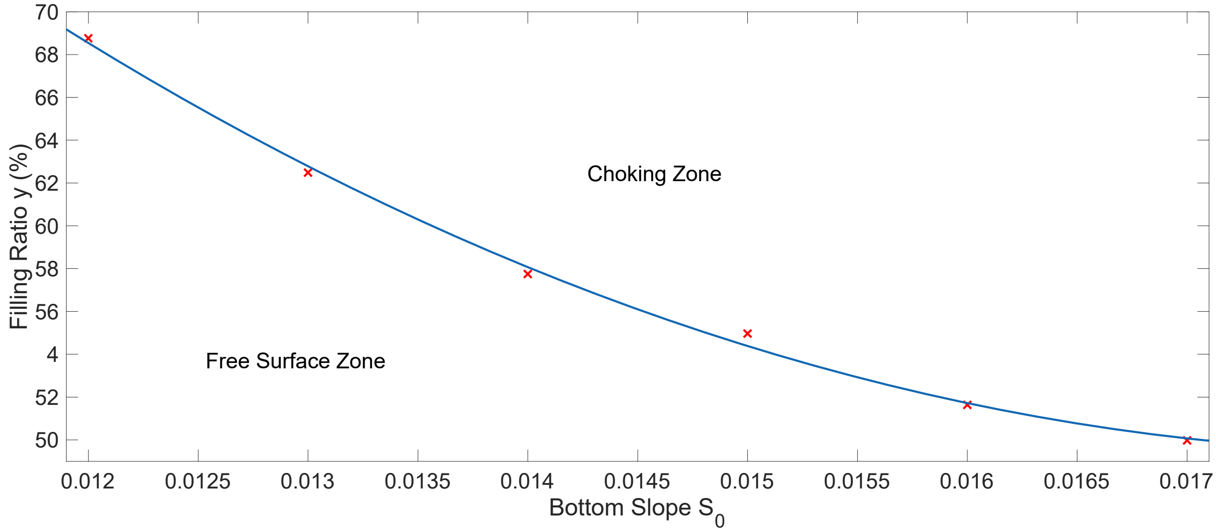

Figure 8. Choking filling ratio for an undular hydraulic jump.

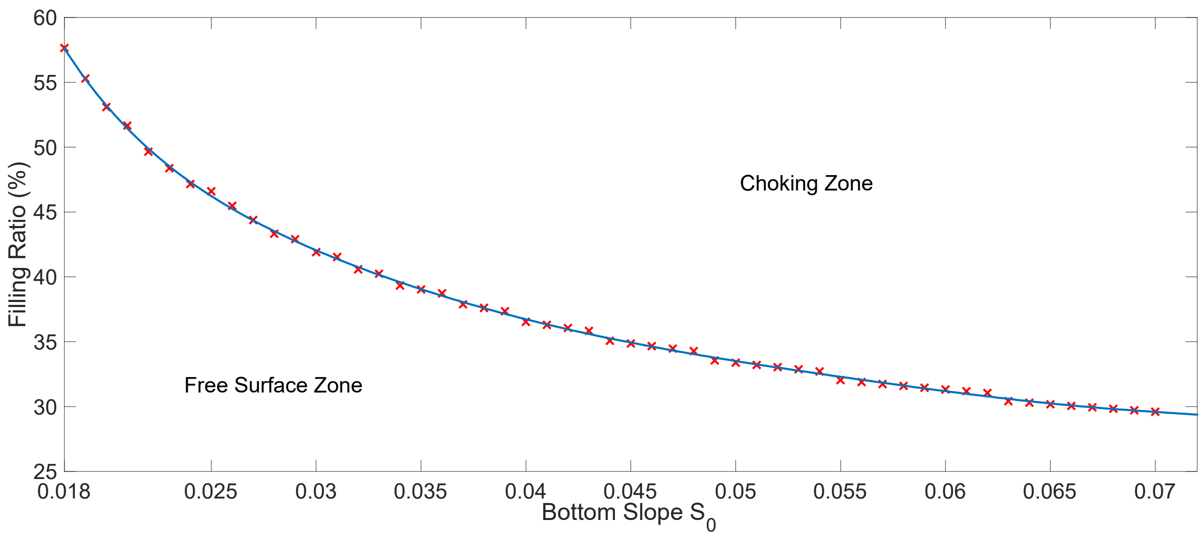

Figure 9. Choking filling ratio for a direct hydraulic jump.

As seen from Figures 8 and 9, the maximum filling ratio for free surface flow to exist downstream of a hydraulic jump in a sewer decreases significantly with increasing bottom slope S_o. This means that the steeper the sewer, the higher the risk for choking, as expected. The regions above the curves in Figures 6 and 7 represent the condition that the downstream flows after a hydraulic jump would become pressurized flows.

Such condition is illustrated in Figure 10. The condition is problematic, but has received little attention in the current practice of designing storm sewers. The current practice is that sewers are normally designed for 75%~85% filling ratio, regardless of whether the slopes are mild or steep. In fact, flows in steep sewers may become pressurised caused by hydraulic jumps before reaching 75% filling ratio. In order for free surface flow to be maintained, the upper bound of design filling ratio for steep sewers needs to be reduced appropriately.

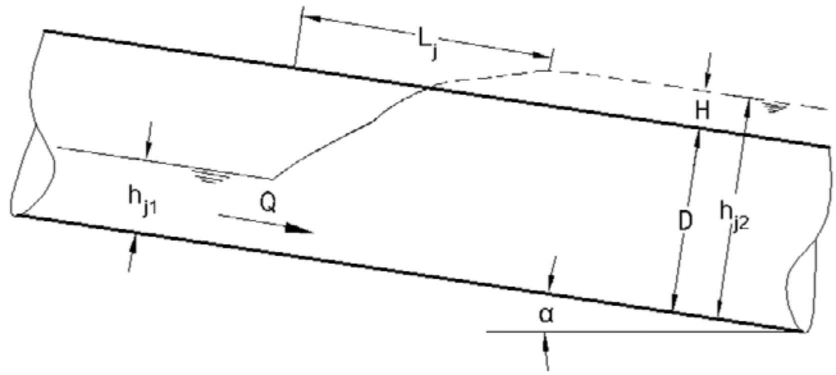

Figure 10. Occurrence of a hydraulic jump in a sloping sewer with pressurized downstream flow, where *H* is the hydrostatic pressure head against the top of the conduit (Montes,1998).

4 CONCLUSION

In this paper, hydraulic jumps in steep storm sewers of circular shape were theoretically analysed. The sewer pipes are assumed to have a diameter of d_o = 0.6 m, and a Manning's roughness factor of n = 0.013. In order to clarify flow patterns and hydraulic types for various bottom slope, values of the Froude number in correspondence to increasing slopes from $S_o = 0.005$ to 0.07 were determined. According to the approach Froude number, there were two main types of hydraulic jumps: For $0.005 \leq S_o \leq 0.017$, possible jumps are mainly an undular hydraulic jump. For $0.017 < S_o \leq 0.07$, possible jumps are a direct hydraulic jump. The depth, h_{1c}, below the first wave crest for an undular hydraulic jump and the sequent depth, h_{j2}, for a direct hydraulic jump were calculated. Flow choking occurs when $h_{1C} \geq d_o$ or $h_{j2} \geq d_o$. This paper presents useful curves of the maximum filling ratio for free surface flow downstream of a hydraulic jump. These curves delineate the regions where the downstream flows would become pressurized flows, which should be avoided in the design of storm sewers. It have been shown that the common design standard for filling ratio should not be applied to steep sewers directly, because of the risk of flow choking caused by hydraulic jumps.

Acknowledgements

This study received financial support from the Natural Sciences and Engineering Research Council of Canada through Discovery Grants held by S. Li and from the Faculty Research Support from Concordia University.

References

ASCE. 1982. *Gravity sanitary design and construction*, American Society of Civil Engineers, New York.

Chow, V. T. 1959. *Open-channel hydraulics*, McGraw Hill, Caldwell, NJ, USA.

Gargano, R., Hager, W.H. 2002. Undular hydraulic jumps in circular conduits. *Journal of Hydraulic Engineering*, 128(11): 1008–1013.

Lowe, N.J., Hotchkiss, R.H., E, James Nelson. 2011. Theoretical Determination of Sequent Depths in Closed Conduits. *Journal of irrigation and drainage engineering*, 137(12): 801-810.

Hager, W.H. 1999b. *Wastewater Hydraulics. 2nd ed.*, Springer, Heidelberg, Berlin.

Montes, J.S., Chanson, H. 1998. Characteristics of undular hydraulic jumps: Experiments and analysis. *Journal of Hydraulic Engineering*, 124(2): 192–205.

Stahl, H., Hager, W.H. 1999a. Hydraulic jump in circular pipes. *Can. J. Civ. Eng.*, 26: 368-373.