PREDICTING THE SHEAR RESISTANCE OF SFRC BEAMS

Cohen, Michael[1,3] and Potapenko, Stanislav[2]

[1] University of Waterloo, Canada

[2] University of Waterloo, Canada

[3] mcohen@uwaterloo.ca

Abstract: This paper presents a literature review on the shear behaviour of steel fiber reinforced concrete (SFRC) and proposes equations for predicting the shear resistance of SFRC beams. First a large database of test results is used to evaluate the effect of various parameters on the shear response of SFRC beams. In addition, the paper reviews various equations proposed in the literature to predict the shear capacity of SFRC beams. The paper then presents equations which can be used to predict the shear resistance of SFRC beams. The shear resistance equations modify the general method of the 2004 CSA A23.3 Standard to account for the effect of steel fibers on shear capacity. The method proposed in this paper and equations proposed in the literature are used to predict a large database of SFRC beam test results. The results show that the method presented in the paper provides reasonably accurate predictions of shear capacity for beams having a wide range of properties.

1   INTRODUCTION

the Catastrophic failures such as the collapse of large RC beams in the 1950s at US Air-force warehouses, as well as more recent failures such as the collapse of the "de La Concorde Bridge" have emphasized the importance of properly accounting for shear in the design of reinforced concrete structures. To prevent shear failure, beams are traditionally reinforced with transverse shear reinforcement in the form of stirrups. One alternative to this reinforcement may lie in the use of steel fiber reinforced concrete (SFRC). Extensive research has shown that the addition of randomly oriented steel fibers enhances the properties of traditional concrete including tensile capacity, fracture toughness and crack control. These improvements attribute to the influence of the randomly orientated fibers in arresting cracks and the resulting enhancements in the post-cracking resistance of the concrete. In beams that are shear-critical, SFRC enhances the diagonal tension capacity of concrete and thus can result in significant enhancements in shear capacity. If added in sufficient quantities, the use of steel fibers can replace traditional transverse reinforcement and promote flexural failure and ductility.

2   PREVIOUS RESEARCH ON THE BEHAVIOUR OF SFRC BEAMS

Extensive research over the past three decades has shown that steel fibers can be used to enhance shear resistance in reinforced concrete beams. Adding steel fibers to a concrete beam without transverse reinforcement improves shear behaviour due to the ability of SFRC to resist and redistribute diagonal tensile stresses after cracking. In addition, the fiber crack-bridging capabilities delay and control the development of cracks. Thus, very much like traditional transverse reinforcement, the use of fibers results in an increase in the overall shear resistance of the beam and can promote flexural failure and ductility.

Para-Montesinos (2006) presented a large database of SFRC beam test results which included results from 16 studies. The database included test results from 147 SFRC beams and 45 companion RC beams having effective depths (*d*) ranging from 180-570 mm, shear-span to depth ratio (*a/d*) ranging from 1.0-6.0, compressive strength (f_c') between 18-104 MPa, longitudinal reinforcement ratios (ρ) ranging from 0.4%-5%, fiber volume fractions (V_f) ranging from 0.25%-0.75% and having straight, hooked or crimped fibers. The results showed that all SFRC beams in the database having $V_f \geq 0.5\%$ exhibited shear stress at failure greater than $0.17\sqrt{f_c'}$, equivalent to V_c as defined in the ACI-318 code (ACI Committee 318 2008) with beams having $V_f \geq 0.75\%$ failing at shear stresses not less than $0.3\sqrt{f_c'}$. Based on the analysis of the database, Parra-Montesinos recommended that steel fibers, if added in sufficient quantity, can be used as a replacement to minimum shear reinforcement in beams subjected to shear forces equivalent to $0.5V_c(0.085)\sqrt{f_c'}$ and $V_c(0.17)\sqrt{f_c'}$. In addition to material performance criteria, Parra-Montesinos also recommended that minimum steel fiber content, V_f, of 0.75% should be provided in order to achieve adequate shear response. This minimum fiber content has also been recommended previously by ACI subcommittee 318-F. Based on the experimental evidence reported in the literature, the use of steel fibers in flexural members is now permitted in several international design codes (see next section).

3   REVIEW OF PREDICTIVE EQUATIONS AND DESIGN GUIDELINES FOR SFRC BEAMS

Over the years, several researchers have proposed equations for predicting the shear resistance of SFRC beams. Examples of models include those proposed by (Khuntia et al. 1999), (Imam et al. 1995), (Ashour et al. 1992), (Mansur et al. 1986), Sharma (1986) and others. As shown in Table 1&2 most of these models can be broadly classified into two categories. The first category of models assumes that fibers improve the concrete shear resistance directly due to the improvement in tensile post-cracking resistance. These models use material tests to quantify the improvement in shear capacity. The second category considers that fibers provide a separate shear contribution that is additional to the concrete shear contribution.

Most of these models are from empirical formulae derived from regression analysis of limited experimental data. Therefore the results obtained from these models can't be relied on in many cases to evaluate the shear capacity in SFRC beams (Minelli 2005). In particular, it is noted that the model proposed by Sharma (1986), and initially recommended by ACI Committee 544, shows very poor accuracy.

Table 1: Various models to predict shear resistance of SFRC Beams (Category 1)

Category 1					
Model	Fiber contribution is:		Fiber effect accounted for using:		
	included in concrete contribution	separate term (additional to concrete contribution)	Split cylinder strength (or MOR)	Fracture mechanics/ Bending test	Fiber factor
(Sharma 1986) $\upsilon_u = kf_t\left(\dfrac{d}{a}\right)^{0.25}$	✓		✓		
(Li et al. 1992) $\upsilon_u = 1.25 + 4.7\left[(f_f f_{sp})^{\frac{3}{4}}\left(\rho\dfrac{d}{a}\right)^{\frac{1}{3}} d^{-\frac{1}{3}}\right]$	✓		✓		
(Cassanova 1996) $V = 0.9bd \cdot \overline{\sigma}_p(w) + V_s$	✓			✓	
(Minelli and Plizzari 2006) $\upsilon_u = \left[\dfrac{0.18}{\gamma_c}k\left(100\rho_1\left(1+2.5\dfrac{f_{eq(0.6-3)}}{f_{ct}}\right)f_{ck}\right)^{1/3} + 0.15\sigma_{CP}\right]$	✓			✓	

Table 2: Various models to predict shear resistance of SFRC Beams (Category 2)

Model	Fiber contribution is:		Fiber effect accounted for using:		
	included in concrete contribution	separate term (additional to concrete contribution)	Split cylinder strength (or MOR)	Fracture mechanics/ Bending test	Fiber factor
(Imam et al. 1995) $$\upsilon_u = 0.6\Psi\sqrt[3]{\omega}\left[\left(f_c\right)^{0.44} + 275\sqrt{\frac{\omega}{\left(a/d\right)^5}}\right]$$		✓			✓
(Mansur et al. 1986) $$\upsilon_u = \left(0.16\sqrt{f_c'} + 17.2\frac{\rho V d}{M}\right) + 0.41\left(\tau \cdot V_f \frac{L_f}{D_f}\right)$$		✓			✓
(Narayanan and Darwish 1987) $$\upsilon_u = e\left(0.24 f_{spfc} + 80\rho\frac{d}{a}\right) + 0.41\left(\tau \cdot V_f \frac{L_f}{D_f} d_f\right)$$		✓			✓
(Ashour et al. 1992) $$\upsilon_u = \left(0.7\sqrt{f_c'} + 7 \cdot V_f \frac{L_f}{D_f} d_f\right)\frac{d}{a} + 17.2\rho\frac{d}{a}$$		✓			✓
(Swamy et al. 1993) $$\upsilon_u = \upsilon_c + 0.37\left(\tau \cdot V_f \frac{L_f}{D_f}\right)$$		✓			✓
(Shin et al. 1994) $$\upsilon_u = 0.19 f_{sp} + 93\left(\rho\frac{d}{a}\right) + 0.34\left(\tau \cdot V_f \frac{L_f}{D_f} d_f\right)$$		✓			✓
(Khuntia et al. 1999) $$\upsilon_u = \left(0.167\alpha + 0.25 \cdot V_f \frac{L_f}{D_f} d_f\right)\sqrt{f_c'}$$		✓			✓
(Kwak et al. 2002) $$\upsilon_u = 3.7 e f_{spfc}^{2/3}\left(\rho\frac{d}{a}\right)^{1/3} + 0.8\left(0.41\tau V_f \frac{L_f}{D_f} d_f\right)$$		✓			✓

It is noted that the majority of the models outlined previously account for the fiber contribution empirically using a "fiber factor", F, which is a function of fiber volume fraction (V_f), diameter (D_f) and length (L_f), as shown in Equation 1, and an additional term to scale this factor based on fiber typology. Nevertheless, the majority of the equations are empirical and do not account for important factors affecting the shear resistance of SFRC beams. As discussed by Minelli (2005).

[1] $F = V_f \frac{L_f}{D_f}$

4 PROPOSED MODEL FOR PREDICTING THE SHEAR RESISTANCE OF SFRC BEAMS

This section describes a procedure for predicting the shear resistance of SFRC beams without web reinforcement. As noted previously, several existing models and design guidelines consider the fiber contribution to shear capacity to be additional to the concrete contribution, much in the same way stirrups improve shear capacity. Other models propose that the fibers should modify the concrete contribution directly, In the present model, the former approach is used and the shear resistance of an SFRC beam is assumed to be equivalent to the expected shear strength of a traditional reinforced concrete beam (V_{no}) plus the additional shear resistance provided by the fibers (V_{fib}) as shown in Equation 2:

[2] $V_{nf} = V_{no} + V_{fib}$

4.1 Reinforced Concrete Contribution to Shear Resistance

In order to compute the reinforced concrete (RC) contribution to shear resistance, V_{no}, the general shear design method of the 2004 CSA Standard can be used (see Equation 3). This method which is based on the Modified Compression Field Theory (MCFT) has been shown to provide accurate estimates of the shear resistance of traditional RC beams.

[3] $V_{no} = V_c + V_s$

where, V_c and V_s are the concrete and transverse steel contributions to shear resistance, respectively. The concrete contribution, V_c, is computed using Equation 4.

[4] $V_c = \emptyset_c \beta \sqrt{f_c'} b_w d_v$

It should be noted that the concrete contribution to shear capacity is generally supported by aggregate interlocking across tension-shear cracks, dowel action and the tensile stress field that becomes militarized in concrete through reinforcement-concrete bond.

In case of using transverse steel reinforcement, its contribution, V_s, can be calculated using Equation 5.

[5] $V_s = \frac{\emptyset_s A_{vf} f_{yh} d_v cot\theta}{s}$

where $b_w d_v$ is the effective shear area represented by the web width multiplied by the effective shear depth of the beam. The factor $\emptyset_c$ is a material reduction factor for concrete (can be taken as 1.0 for the purpose of analysis). The value β accounts for the ability of the concrete to transmit tensile stresses between the cracks. The angle θ is the angle of inclination of the diagonal compressive stresses to the longitudinal strain at mid-depth of the cross section, ε_x, as shown in Equation 6 and Equation 7, respectively.

[6] $\beta = \frac{0.40}{1+1500\varepsilon_x} \times \frac{1300}{1000+s_{ze}}$

[7] $\theta = 29^o + 7000\varepsilon_x$

The longitudinal strain at mid-depth of the cross section, ε_x, is computed using Equation 8 for the case of moment and shear.

[8] $\varepsilon_x = \dfrac{\left(\frac{M}{d_v}\right)+V}{2\times E_s \times A_s}$

To determine the shear strength, the shear, V, should correspond to the expected maximum shear resistance of the beam. The moment, M, represents the corresponding moment at the critical section in the beam (for the beams in this analysis the critical section is taken at a distance d_v from the loading point). A_s and E_s are the cross-sectional area and modulus of elasticity of the longitudinal tension reinforcement, respectively.

4.2   Pullout Strength of Hooked-End Fibers

Studies have shown that anchorage properties of fibers play an important role in improving the pullout resistance of hooked-end steel fibers. Alwan et al. (1999) suggested that the mechanical contribution of the hook,$\Delta P'$, is a function of the cold work needed to straighten the fiber as it is being pulled out from the matrix. The simplified expression for $\Delta P'$ is shown in Equation 9:

[9] $\Delta P' = \dfrac{3.05}{\cos(45^0 \times \pi/180^0)}\left(f_{fy}\times\dfrac{\pi(D_f/2)^2}{6}\right)$

The value f_{fy} is the fiber yield strength (in psi) and D_f is the fiber diameter (in inches). As an example for a typical Dramix ZP305 fiber, $\Delta P'$ is found to be 34 lbs (152 N).

Therefore, in the case of hooked-end fibers, taking into account the hook contribution, an estimate of the pullout strength, $F_{pullout}$, can be made using Equation 10, where the contribution of the hook ($\Delta P'$) is added to the load needed to cause debonding of the fiber:

[10] $F_{pullout} = \left(\tau_{bond}\times\pi\times D_f \times \dfrac{L_{f,straight}}{2}\right) + \Delta P'$

The parameter $L_{f,straight}$ is the length of the straight portion of the fiber.

4.3   Pullout Strength of Crimped Fibers

In order to take into account the enhancement in pullout strength provided by the deformed shape of a crimped fiber, the present model suggests using a "deformation contribution factor", α_m which scales the expected pullout strength of a straight smooth fiber as shown in Equation 11. Using data on the pullout strength of crimped fibers in the literature, α_m was taken as the average of the ratio of the bond strength of the deformed fiber (taken as measured pullout load divided by the surface area of the fiber) and the expected bond strength of a straight smooth fiber. Based on this analysis, α_m was estimated to be 2.84 for the case of crimped fibers.

[11] $F_{pullout} = \tau_{bond}\times\pi\times D_f\times\dfrac{L_f}{2}\times\alpha_m$

In the present work the simplified formula shown below was developed to estimate the bond-shear strength, τ_{bond}:

[12] $\tau_{bond} = 1.11\times(f_{ct})^{1.354}$

where f_{ct} is the tensile strength of concrete and can be found using the following equation:

[13] $f_{ct} = 0.33\sqrt{f_c'}$ (MPa)

4.4   Fiber Contribution to Shear Resistance

In order to account for the influence of the fibers on shear strength one can examine the free-body diagram of an SFRC beam with a crack inclination θ (see Figure 1). The fiber contribution to shear resistance, V_{fib}, can be related to the pullout strength of the fibers crossing this cracking plane as shown in Equation 14:

[14] $V_{fib} = \psi \times K \times (N_{fibers} \times \alpha_v F_{pollout}) \times b_w d_v cot\theta$

The effective number of fibers per unit area, N_{fibers}, for fibers randomly oriented in three dimensions can be calculated using Equation 15:

[15] $N_{fibers} = \frac{V_f}{A_f} \times \alpha_\theta \times \eta_1$

where A_f is the cross-sectional area of the fiber, and where V_f is the volume fraction of fibers in the matrix. The orientation factor, α_θ, is used to account for the random orientation of the fibers crossing any arbitrary cracking plane and is taken as 3/8. The length factor η_1 is used to account for the variability in the fiber embedment length across the cracking plane and is taken as 0.5.
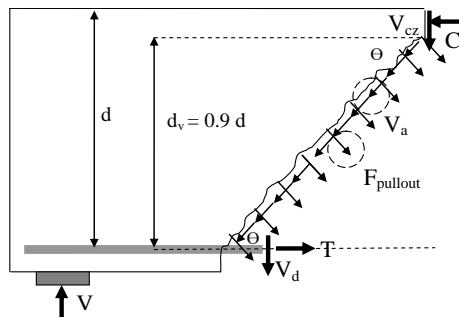
Figure 1: Fiber pullout resistance contributing to shear resistance. (Cohen 2012)

The pullout strength, $F_{pullout}$, is valid for the situation of pure tensile pullout. However in a beam, the fibers will resist tension along the diagonal cracks while undergoing a shear deformation (see Figure 2). Hence, in the proposed model it is assumed that the fibers will have reduced pullout efficiency in the situation of combined tension and shear. Using regression analysis, an empirically determined modification factor "Pullout Reduction Factor" of 0.8, α_v, was used to reduce the pullout resistance of the fibers (the pullout reduction factor was previously proposed in the model of Aoude (2008).
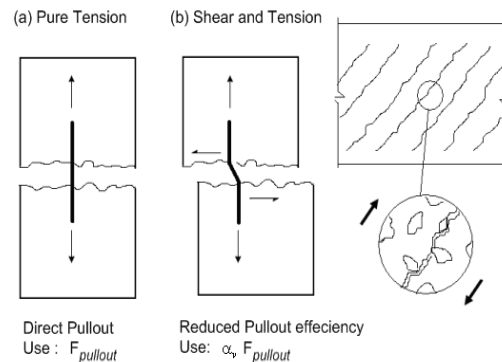

Figure 2: Reduced pullout resistance in the case of combined tension and shear. (Aoude 2008)

Previous studies have shown that the beam's shear span-to-effective depth ratio (*a/d*) has a significant influence on the shear resistance of SFRC beams. In the proposed model a factor, K, is used to account

for the effect of the (*a/d*) ratio as shown in Equation 16. This expression was determined based on regression analysis using a large database of experimental results from the literature.

[16] $K = 1.72 \times \dfrac{d}{a}$

Figure 3(a) shows the relationship between normalized shear stress and *a/d* ratio from four different studies in the literature. It can be seen that as the *a/d* ratio increases, the normalized shear stress reduces. It can also be seen that at an *a/d* ratio of 2.0, the normalized shear strength of the beams approaches a value of 0.75. Assuming that the fiber participation is full at an *a/d* ratio of 2.0 and reduces beyond the point, Figure 3(b) plots the relationship of normalized shear strength divided by 0.75 as a function of *a/d* ratio. It can be seen that the empirically-derived expression for K gives a reasonably good correlation to the experimental data reported in the literature.

Normal shear stress as a function of *a/d* ratio

Correlation between normalized data and participation factor, K.
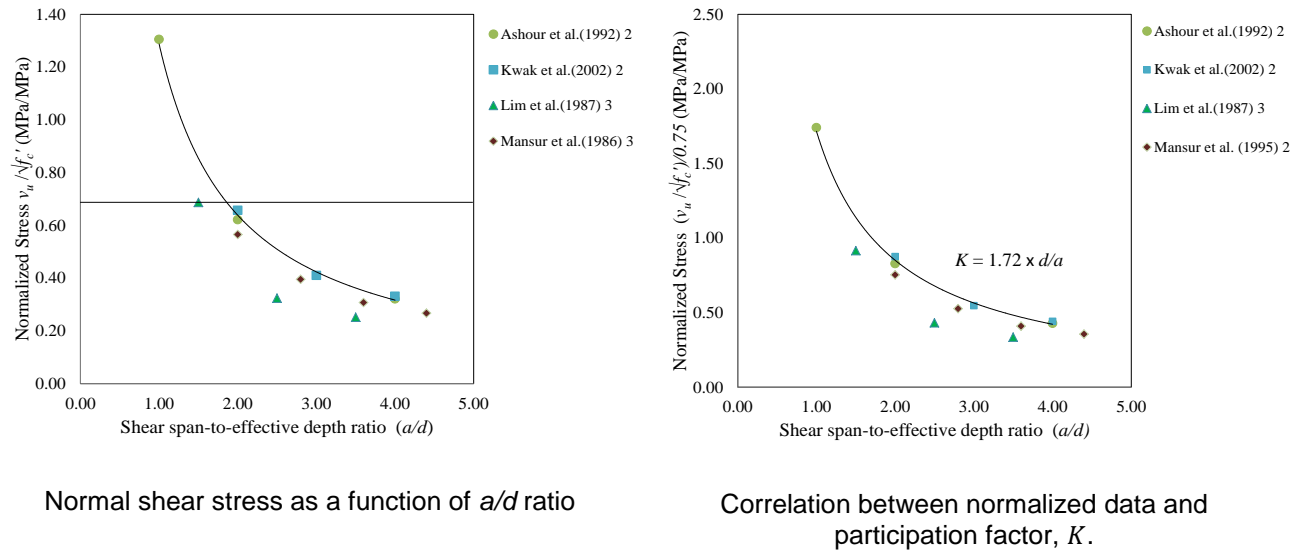
Figure 3: the calculation of the participation factor, K. (Cohen 2012)

Research has shown that the shear stress at failure in RC beams without web reinforcement reduces as the size increase. Some researchers have postulated that this "size effect" is eliminated in SFRC beams. Although there is very limited data on the shear behaviour of larger SFRC beams, various studies in the literature indicate that there may be a size effect in larger SFRC beams. In the proposed model this size effect is taken into account using the size effect factor, ψ. The expression shown in Equation 17 is the same expression suggested by Zararis and Papaakis (2001) for accounting for size effect in traditional RC beams:

[17] $\psi = 1.2 - 0.2 \dfrac{a}{d} d$

5   PREDICTION OF SHEAR STRENGTH OF SFRC BEAMS

In this section, the models proposed by other researchers and the author are used to predict the shear capacities of a large database of SFRC beams from the literature. The database includes the results from 173 SFRC beams (120 beams with hooked-end fibers and 53 with crimped fibers), including 137 beams that failed in shear. It should be noted that many of the experimental tests available in the literature present the results of beams that have small *a/d* ratio. Such beams carry load by strut and tie action and in such cases the strength of the beams is strongly influenced by the details near the supports (Collins and Mitchell 1997). From the database of 173 beam test results taken from the literature, 129 beams have *a/d* ratio greater than 2.3 (with 97 beams in this category failing in shear). Figure 4 shows the predictions of the

beams using four different models proposed in the literature and considering all 173 beams in the database. It is very evident that the majority of the models proposed in the literature show high scatter.

Finally, Figure 5(a) and Figure 6(a) compare the experimental shear capacities of all 173 beams in the database to those predicted using the analytical models for hooked-end and crimped fibers proposed in this paper. Figure 5(b) and Figure 6(b) compare the experimental and predicted shear capacities of only those beams with *a/d* ratio greater than 2.3. One can see that the predictions using the models proposed in the present study agree reasonably well with the actual capacities for the majority of the beams for the case of slender beams.

6   Conclusion

Steel Fiber Reinforced Concrete is a promising material which offers many advantages that can be adopted for structural applications. This paper proposed a reliable method to predict the shear strength of SFRC beams. Based on an evaluation of a large database of results, existing equations in the literature are empirical and many do not accurately predict the shear resistance of SFRC beams, there is thus a need to develop reliable and accurate prediction equations. An analytical model which can predict the ultimate shear resistance of SFRC beams was proposed. The model accounts for: 1) the effect of fiber deformation on pullout strength (with equations developed for both hooked end and crimped steel fibers); 2) the influence of bond-shear strength on pullout strength (with an equation which computes bond strength based on matrix strength); 3) the influence of shear span-to-depth ratio (*a/d*) on the shear strength of SFRC beams (with a factor that accounts for *a/d* ratio); 4) the influence of beam size on the shear strength of SFRC beams (with a factor that accounts for the effect of increased beam size).
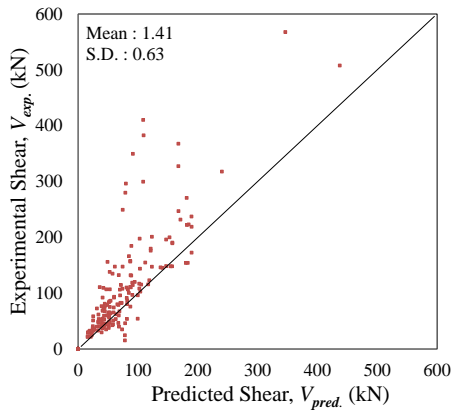
The results show that the analytical model provides accurate and reliable predictions for a large database of SFRC beam test results reported in the literature. The model also provides reasonable predictions for SCFRC beams.
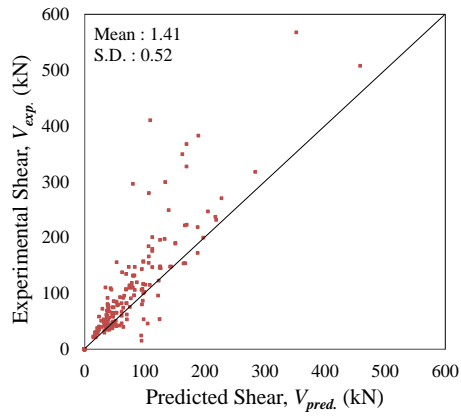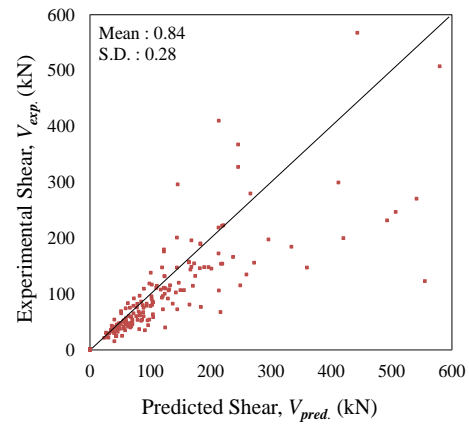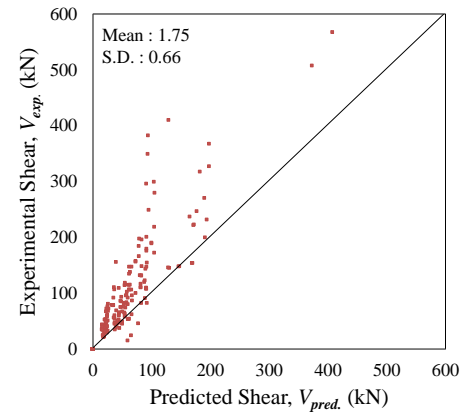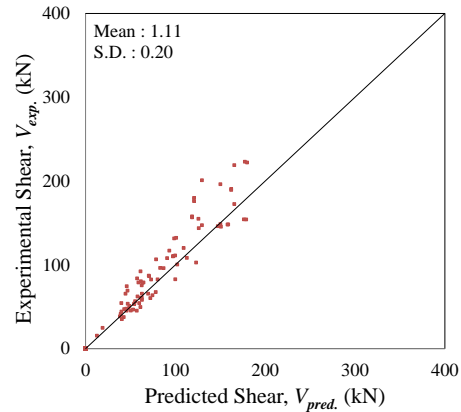
Figure 4: Experimental vs. predicted shear capacities using equations suggested by other researchers and including all beams in the database. (Cohen 2012)
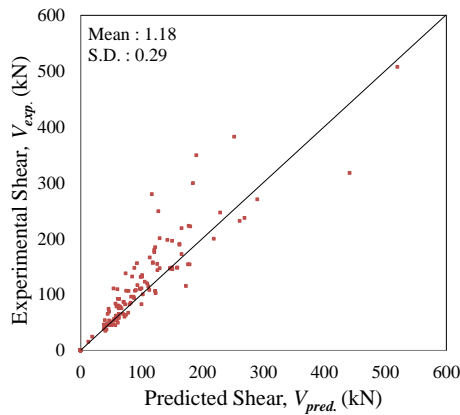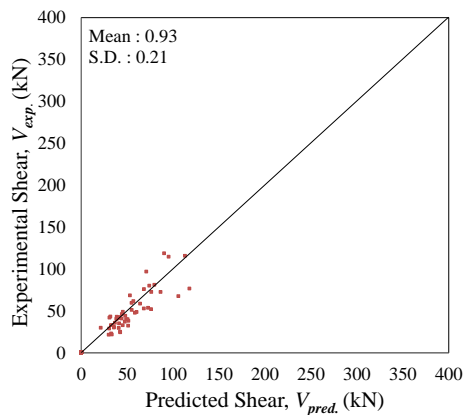
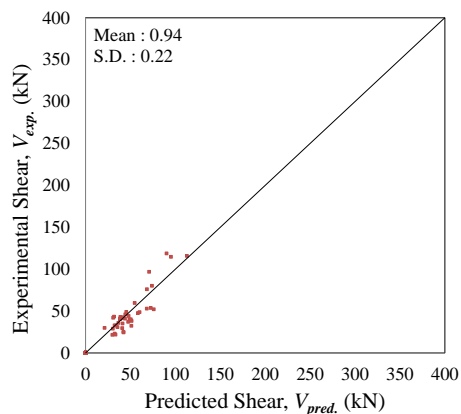(a) Experimental vs. predicted shear capacities of all the beams

(b) Experimental vs. predicted shear capacities of beams with *a/d* > 2.3

Figure 5: Experimental vs. predicted shear capacities using the proposed model for hooked-end fiber. (Cohen 2012)

(a) Experimental vs. predicted shear capacities of all the beams

(b) Experimental vs. predicted shear capacities of beams with *a/d* > 2.3

Figure 6: Experimental vs. predicted shear capacities using the proposed model for crimped fiber. (Cohen 2012)

7 References

ACI Committee 318. 2008. *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary.* Farmington Hills, Michigan, USA: Ammerican Concrete Institute, 473.

Alwan, J. M., A. E. Naaman, and P. Guerrero. 1999. "Effect of Mechanical Clamping on the Pullout Response of Hooked Steel Fibers Embedded in Cementitious Matrices." *Concrete Science and Engineering* **1** (1): 15-25.

Aoude, H. 2008. *Structural Behaviour of Steel Fiber Reinforced Concrete Members.* PhD Thesis, McGill University, Montreal, Canada: Department of Civil Engineering and Applied Mechanics.

Ashour, S. A., G. S. Hasanain, and F. F. Wafa. 1992. "Shear Behaviour of High-Strength Fiber Reinforced Concrete Beams." *ACI Structural Journal* **89** (2): 176-184.

Cassanova, P. 1996. *Bretons renforces de fibers metalliques: du materiau a al structure. Etude experimentale et analyse du comportement de poutres soumises a la flexion et a l'effort tranchant .* Paris, France: Materiaux, Ecole Nationale des Ponts et Chaussees.

Cohen, M. 2012. *Structural Behaviour of Self Consolidating Steel Fiber Reinforced Concrete Beams.* Master's Thesis, Civil Engineering Department, Ottawa, Canada: University of Ottawa.

Collins, M. P., and D. Mitchell. 1997. "*Prestressed Concrete Structures.*" Toronto and Montreal, Canada: Response Publication.

Imam, M., L. Vandewalle, and F. Mortelmans. 1995. "Shear-Moment Analysis of Reinforced High Strength Concrete Beams Containing Steel Fibers." *Canadian Journal of Civil Engineering* **22** (3): 462-470.

Khuntia , M., B. Stojadinovic, and S. C. Goel. 1999. "Shear Strength of Normal and High-Strength Fiber Reinforced Concrete Beams without Strirrups." *ACI Structural Journal* **96** (2): 282-289.

Kwak, Y. K., M. O. Eberhard, W. S. Kim, and J. Kim. 2002. "Shear Strength of Steel Fiber-Reinforced Concrete Beams Without Stirrups." *ACI Structural Journal* **99** (4): 530-538.

Li, V. C., R. Ward, and A. M. Hamza. 1992. "Steel and Synthetic Fibers as Shear Reinforcement." *ACI Materials Journal* **89** (5): 499-508.

Mansur, M. A., K. C. Ong, and A. P. Paramasivam. 1986. "Shear Strength of Fibrous Concrete Beams without Stirrups." *ASCE Journal of Structural Engineering* **112** (9): 2066-2079.

Minelli , F., and G. A. Plizzari. 2006. "Steel Fibers as Shear Reinforcement for Beams." *Proceedings of The Second Fib Congress.* Naples, Italy. 12.

Minelli, F. 2005. *Plain and Fiber Reinforced Concrete Beams under Shear Loading.* PhD Thesis, Department of Civil Engineering, Italy: University of Brescia.

Narayanan, R., and I. Y. Darwish . 1987. "Use of Steel Fibers as Shear Reinforcement." *ACI Structural Journal* **84** (3): 216-227.

Para-Montesinos, G. J. 2006. "Shear Strength of Beams with Deformed Steel Fibers." *Journal of American Concrete Institute* 28 (11): 57-66.

Sharma, A. K. 1986. "Shear Strength of Steel Fiber Reinforced Concrete Beams." *Journal of American Concrete Institute* **83** (4): 624-628.

Shin, S. w., J. G. Oh, and S. K. Ghosh. 1994. "Shear Behaviour of Laboratory-Sized High-Strength Concrete Beams Reinforced with Bars abd Steel Fibers." *ACI Special Publication* **142**: 181-200.

Swamy, R. N., R. Jones, and A. T. Chiam. 1993. "Influence of Steel Fibers on the Shear Resistance of Lightweight Concrete I-Beams." *ACI Structural Journal* **90** (1): 103-114.

Zararis , P. D., and G. C. Papaakis. 2001. "Diagonal Shear Failure and Size Effect in RC Beams without Web Reinforcement." *ASCE Journal of Structural Engineering* **127** (7): 733-742.