



INFLUENCE OF DELAY RISK SCORE ON ALLOCATING FLOAT TO ACTIVITIES IN NETWORK SCHEDULES

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ABSTRACT

Float and buffers are often used by project planners to cushion delays. Float is generated objectively as an output of the critical path method (CPM), while a buffer is created subjectively by the planner. How to fairly allocate float or buffers at the activity level has attracted increasing attention from numerous researchers. This research is inspired by existing political apportionment to allocate float based upon activities' delay risk score (DRS) combined with their criticality index (CI) from a Monte Carlo (MC) simulation. Its contribution to the body of knowledge is threefold: First, it creates an area performance index and varies the exponent to assess delay trends for individual activities and the entire project. Second, it explores activities with small and large DRS. Third, it evaluates the area performance index to find the best combination. It sheds new light on the classic question of how float can best benefit a project.

Keywords: Schedules, delay risk score, float, apportionment methods, simulation, performance

1. INTRODUCTION

A delay can be defined as “an act or event that extends the time required to perform tasks under a contract” (Stumpf 2000, p. 32). Scheduling construction projects must anticipate and prevent delays, because an ideal scenario wherein each activity runs from its planned early start to its early finish time rarely if ever exists (Arcuri and Hildreth 2007). Such an uncertain environment will randomly vary durations within a range. For simulation purposes, a probability distribution function (PDF) is selected for an activity's duration, e.g., beta, triangular, or uniform distribution (Hajdu and Bokor 2016, Fente *et al.* 2000). When faced with random delays, float or buffers provide “a cushion or shield against the negative impact of disruptions and variability” (Russell *et al.* 2014, p. 2). Float and buffers “are two closely related phenomena that only differ in whether they are intentional” (Lucko *et al.* 2016, p. 750). Allocating float means total float (TF), contract float, etc., while buffers include a project buffer (PB) and feeding buffers (FB) that are generated by critical chain project management (CCPM), as well as time contingency under the contract (Su *et al.* 2017b). A by-product of CPM (de la Garza *et al.* 1991), TF is only available to noncritical activities, but “[a]ctivities that need flexibility most desperately are called critical because by definition they received none during scheduling” (Su *et al.* 2017a). Similarly, FB in CCPM is just available for activities on noncritical chains in a schedule network. Contract float is the gap between the project finish time and the contract due date. It resides at the project end and resembles PB in CCPM. They are consumed on an unfair and inefficient “a first-come, first-served basis, with the potential to be exhausted before the project is over” (Barraza 2011, p. 259). Thus how to fairly and efficiently allocate float and

buffers to the activity level must be explored. Fortunately, political apportionment research, which is defined as “determining how to divide a given integer number of representatives or delegates proportionally among given constituencies according to their respective sizes” (Balinski and Young 1977, p. 607), has numerous analogies with this float allocation research: 1. Both seek to allocate a valuable limited resource (i.e. seats or float); 2. Both resources are counted in integer units (i.e. seats or days); 3. Both need to fairly and equitably perform said apportionment to subjects (i.e. states or activities) of different sizes. Expanding the aforementioned research that had applied political apportionment to float / buffer allocation, this research examines how delay can be characterized and influences the performance of such float / buffer allocation in schedules with MC simulations.

2. LITERATURE REVIEW

2.1 Float / Buffer Allocation Research

Two perspectives exist on the relation between float and buffer (for brevity, hereinafter it is referred to as ‘float’) ownership and allocation. One treats them as two parallel issues: Ownership is a legal entitlement to float; allocation is a practical division of dividing float among activities regardless of who is deemed to own it – the owner, general contractor (GC), designer (architect or engineer), or subcontractors. But the other view also holds: Ownership as a challenge of fair division, where ‘*who should own float*’ leads to allocating it in a manner that is consistent with the respective entitlement and *vice versa*. Table 1 lists previous attempts in these two categories. Under ownership, ten approaches arrive from various angles. Some take an absolute view – owner ownership, GC ownership, and equal proportion advocate that TF should belong only to the owner or GC, or half-half to each. Treating TF as a commodity further raises the possibility to make it tradable among project participants. Others take a relative view – e.g. the project, bar, contract risk, path distribution, and day-by-day approaches. They attempted a solution for a specific situation of the target schedule or project. The total risk approach is a hybrid that combines four others, including the contract risk, path distribution, day-by-day, and commodity approach. All items in this category focus on TF ownership. But this should be expanded to contract float ownership to enable protecting critical activities.

Table 1: Float Ownership and Allocation Methods (modified after Su et al. 2017a)

Method	Definition
Float/Buffer Ownership	
Owner	Owner approves design and pay; should own TF (Prateapusanond 2003, Pasiphol and Popescu 1995)
GC	GC decides means and methods for execution work, including its schedule, resources; should own TF
Project	Case law vague; both owner and GC should own (Arditi and Pattanakitchamroon 2006, Peters 2003)
Equal Proportion	Contract language remains vague; proposes simplistic 50%-50% split of TF (Prateapusanond 2003)
Bar	(Non-)excusable delay only allowed after consuming ‘bar’ TF, sharing unclear ((Ponce de Leon 1986)
Contract Risk	Contract implies GC owns TF, e.g. if lump sum (Hartman <i>et al.</i> 1997, Householder and Rutland 1990)
Path Distribution	Non-critical activities should own TF proportional to their duration (Ponce de Leon 1982), priority by path length indicating how soon they could become critical themselves (Pasiphol and Popescu 1994)
Day-by-Day	Tracking TF ‘debit’ and ‘credit’ against company delay or acceleration (Al-Gahtani and Mohan 2007)
Commodity	GC has valuable commodity TF, tradable monetary value is cost (LF-EF)/TF (de la Garza <i>et al.</i> 1991)
Total Risk	Hybrid of previous four approaches, TF should be allocated proportional to risk responsibility and could be sold to other participant if they need to consume TF instead of its owner (Al-Gahtani 2006)
Float/Buffer Allocation	
Critical Chain	PB = Original critical main chain duration $\times 0.5 \times 0.5$ FB = Original critical side chain duration $\times 0.5 \times 0.5$
Stochastic	Planned minus target duration is project time allowance; which is sum of activity time allowances, which are maximum minus target duration for given risk percentile as user specifies (Barraza 2011)
Fuzzy	For one risk, fuzzy time contingency is adjusted fuzzy time extension; for multiple risks, it is their combined impact, using the fuzzy maximum operator in calculation (Pawan and Lorterapong 2015)
Political	Apportionment based on activity DRS and CI, simulations with different rounding, exponents, and available contract float, select best combination by its delay protection performance (Su <i>et al.</i> 2017)

Under allocation, all four approaches focus on distributing a buffer or time contingency. The aforementioned CCPM treats the original schedule duration with suspicion of being inflated by 200%. Per Table 1, from this unsubstantiated assumption CCPM cuts all activities original durations in half and then returns half of this cut time as PB at the end of the critical chain. Similarly, it uses a quarter of the original length of noncritical activity durations as FB wherever such a side path merges into the critical chain (Goldratt 1997). The stochastic allocation of project allowance (SAPA) method distributes a project time allowance at the activity level, where the planned critical activity time allowance (ATA_i) is the difference between the maximum allowed duration with a risk level percentile (Dp_i) and the estimated target duration for that critical activity (Td_i), i.e. $TTA = \sum_{i \in cp} (Dp_i - Td_i) = \sum_{i \in cp} ATA_i$ (Barraza 2011). The

fuzzy method distinguishes two cases of allocating a time contingency: First, if the activity is influenced by a single risk, then its time contingency is the adjusted fuzzy time extension. Second, if it is affected by multiple risks, then a combined impact of those risks is calculated as the fuzzy activity time contingency from a fuzzy maximum operator (Pawan and Lorterapong 2015). A recent float allocation method that is inspired by political apportionment has successfully investigated how to protect activities from negative impacts of delays by allocating float to them; and considering how criticality varies by capturing the schedule structure via nonzero CI values (Su *et al.* 2017a, 2017b).

2.2 Political Apportionment Methods

Political apportionment is “the process of allocating seats to geographical units (... constituencies, states, regions)” (Gallagher and Mitchell 2005, p. 631). It assumes a country has a total population P and contains n states. The i^{th} state has a population of p_i ($\sum p_i = P$). Thus a state has p_i / P of the total population. The country’s congress has N total seats and all states should be given seats according to their population. Thus each state theoretically should have $q_i = N \cdot p_i / P$ seats, which is called *fair share*, *standard quota*, or *exact quota* (Niemeyer and Niemeyer 2008).

But one seat cannot be split; each state must have an integer number of seats. This requirement led to apportionment methods to round state quotas into integer seats in a fair manner. Table 2 compares all six common apportionments, including the largest remainder method and five divisor methods (Jones and Wilson 2010). A bias exists in the largest remainder method, which “was uncovered in the 1880’s when the notorious ‘Alabama paradox’ occurred: Hamilton’s method, under the name of Vinton’s method of 1850, was the law, and gave to Alabama 8 seats in a house of 299 seats but only 7 in a house of 300” (Balinski and Young 1978, p. 280). Table 2 also lists algorithms of divisor methods, which use different ways to search a proper divisor: First divide the population or quota of each state by a divisor. Then apply a rounding to make the sum of each state’s seats equal to N . Some of these methods incur a bias to larger states, others to smaller ones. Only the arithmetic mean has no bias (Balinski and Young 2001).

Table 2: Comparison of Apportionment Methods

Method	Inventor	Algorithm	Seats	Bias
Largest Remainder (LR)	Hamilton, Vinton, Hare	1: Give each party $\lfloor q_i \rfloor$ seats 2: Sort remainders $q_i - \lfloor q_i \rfloor$ in descending order 3: Give one more seat to each party until all seats apportioned	$\lfloor q_i \rfloor$ or $\lfloor q_i \rfloor + 1$	Alabama Paradox
Greatest Divisor (GD)	Jefferson, d’Hondt	1: Search divisor $0 < \lambda < N + 1$ 2: Check $\sum \lfloor p_i / \lambda \rfloor = N$	$\lfloor p_i / \lambda \rfloor$	Larger
Smallest Divisor (SD)	Adams	1: Search divisor $0 < \lambda < N$ 2: Check $\sum \lceil p_i / \lambda \rceil = N$	$\lceil p_i / \lambda \rceil$	Smaller
Arithmetic Mean (AM)	Webster, Willcox, Sainte-Laguë, Schepers	1: Arithmetic mean between λ and $\lambda + 1$ $am = (\lambda + (\lambda + 1)) / 2$, note $0 < \lambda < N$ 2: $s_i = \lfloor p_i / am \rfloor$ 3: Check $\sum s_i = N$, adjust divisor λ as necessary	s_i	None
Geometric Mean (GM)	Huntington-Hill	1: Round down each quota $\lfloor q_i \rfloor$ 2: Geometric mean of $\lfloor q_i \rfloor$ and $\lfloor q_i \rfloor + 1$ is $gm_i = (\lfloor q_i \rfloor \times (\lfloor q_i \rfloor + 1))^{1/2}$ 3: If $q_i < gm_i$, $s_i = \lfloor q_i \rfloor$, else $s_i = \lfloor q_i \rfloor + 1$ 4: Check $\sum s_i = N$, adjust q_i by dividing divisor λ as needed	s_i	Smaller
Harmonic Mean (HM)	Dean	1: Round down each quota $\lfloor q_i \rfloor$ 2: Harmonic mean of $\lfloor q_i \rfloor$ and $\lfloor q_i \rfloor + 1$ is $hm_i = 2 / (1 / \lfloor q_i \rfloor + 1 / (\lfloor q_i \rfloor + 1))$ 3: If $q_i < hm_i$, $s_i = \lfloor q_i \rfloor$, else $s_i = \lfloor q_i \rfloor + 1$ 4: Check $\sum s_i = N$, adjust q_i by dividing divisor λ as needed	s_i	Smaller

3 METHODOLOGY

The methodology follows previous research by these authors. It employs the DRS – explained in the next section – and CI to allocate float in analogy to the political apportionment methods (Su *et al.* 2017b). Several innovations are made: Defining this new DRS will allow that the quota of an activity can reflect both the probability and severity of a probabilistic delay. Treating activities with nonzero CI as candidates in the float allocation process can solve the problem of criticality shifting that may occur as schedules are executed and updated. It searches for the optimum combination of apportionment method, exponent, and required contract float. But there still exists a gap in tracking the performance of single activities in a comprehensive manner. Three **Research Objectives** are set as follows:

1. Create an index to evaluate performance trends for a single activity when the exponent of the method is varied;
2. Explore how critical activities with small and large DRS behave in terms of being protected against delays;
3. Evaluate the new area performance indices for different apportionment methods to identify the best approach.

3.1 DRS

In real-world construction projects, longer activities do not necessarily incur serious delays; shorter activities may be delayed strongly. This is because duration alone is not a sufficient proxy for risk, the nature of the work also plays a major role. In other words, severity of a delay may not be strongly positively correlated to duration but to the delay period (or magnitude, i.e. maximum minus mode) and the probability of its occurrence. Suppose two activities have the same delay magnitude, but their delay probabilities are different – one is often delayed, one rarely. The former expected average delay is worse than the latter. Therefore probabilistic delays should be measured as a combination of their magnitude and probability (Su *et al.* 2017b). The value of the DRS will be calculated as the product of these two factors per Equation [1]. It equals the period (i.e. maximum minus mode) multiplied with the area of the delayed portion of the PDF (i.e. the integral of PDF from mode to maximum). Figure 1 shows DRS calculations for different probability distributions, including the triangular, uniform, beta, and normal distributions. Note that the dark areas represent the delayed portions of the respective PDF. Those shaded areas are the delay probabilities of each PDF.

$$[1] \text{ DRS} = \text{Delay Magnitude} \times \text{Delay Probability} = (\text{maximum} - \text{mode}) \times \int_{\text{mode}}^{\text{maximum}} f(x)_{PDF} dx$$

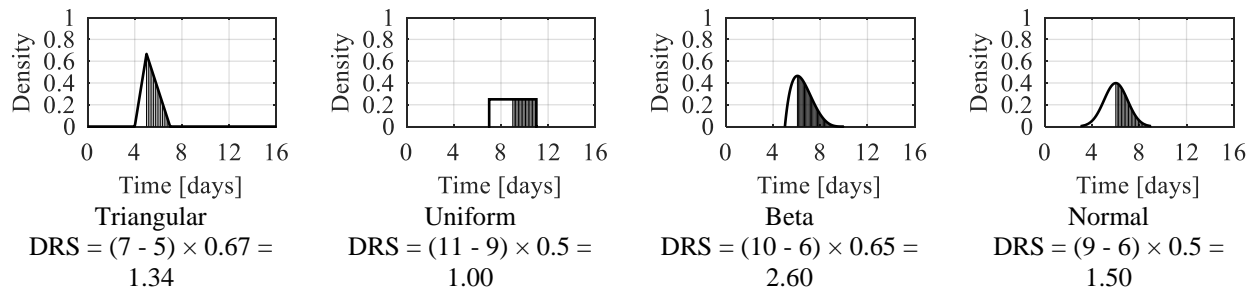


Figure 1: DRS for Different Probability Distributions

3.2 Methodology Flowchart

The float allocation method inspired by political apportionment using DRS and CI is represented in the flowchart of Figure 2, which extends previous research (Su *et al.* 2017a). At the top of the flowchart, it captures schedule data, including activity names, sequential relations, and probability distributions for durations. Then an MC simulation determines CI as the ratio of the number of times that an activity is critical divided by the total number of simulation runs. A network diagram of the schedule can also be plotted to aid a user in understanding the schedule structure.

In the middle of the flowchart, two parallel processes are active: 1. The first step in the float allocation calculates the DRS for all activities with nonzero CI. The counterpart of this step in the political apportionment adds an exponent n to these DRS to calculate quotas for these activities. Note that n varies from 0 to 1 to assess the performance of the apportionment. 2. A value is assigned for contract float (FLOAT) and transmitted to the apportionment in analogy to the total number of seats. Of course FLOAT is an integer. It is incremented from 1 to a large value, here 100 (which exceeds the sum of worst case delays of all critical activities). This allows evaluating when saturation is achieved to protect the schedule from random delays. 3. In this step the political apportionment begins by employing different methods to create solutions for what portion of FLOAT each candidate activity receives as its own. It transmits this as the allocated float s_i to each activity. Of course the sum of all individual s_i must equal FLOAT for any method.

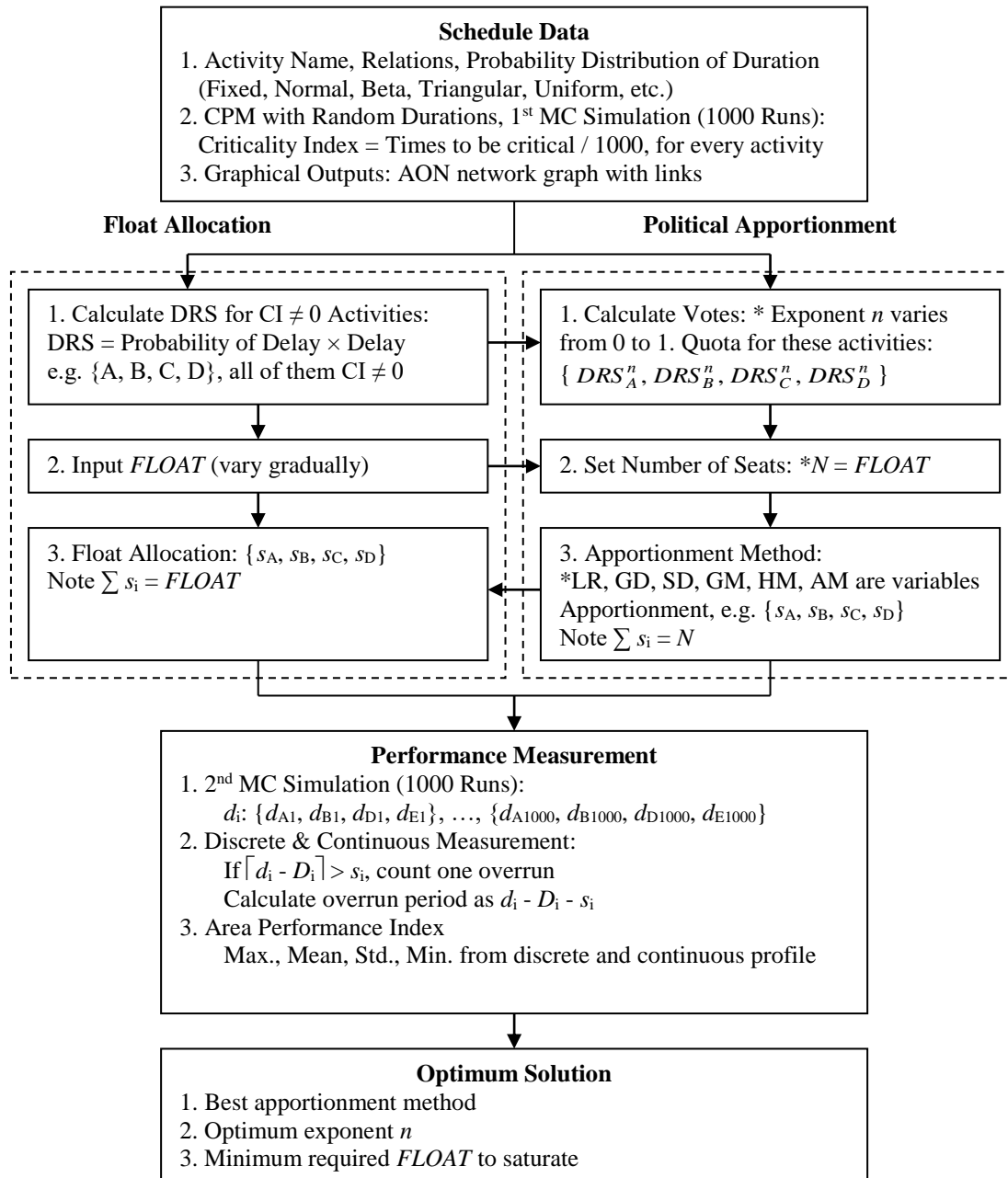


Figure 2: Methodology Flowchart (modified after Su *et al.* 2017b)

At the bottom of the flowchart, another MC simulation randomizes schedules. A discrete measurement is defined as the overrun count if $\lceil d_i - D_i \rceil > s_i$. A continuous measurement is defined as the overrun period $(d_i - D_i - s_i)$. Different from the authors' previous research, this paper creates an area performance index. Four statistical values (maximum, minimum, mean, standard deviation) for it will be calculated for each schedule profile. Comparing then will allow selecting the optimum solution, which will consist of a specific rounding method for apportionment (from those that Table 2 lists), the optimum exponent n , and the minimum required FLOAT to saturate and thus prevent any delays.

4 Area Performance Index

A schedule with 60 activities from the Project Scheduling Problem Library (PSPLIB, Kolisch and Sprecher 1996) is selected for analysis. Figure 3 shows its network with critical activities in gray. Figure 4 contains discrete and

continuous performance profiles for overrun count and overrun period over FLOAT, respectively. Figures 4a and 4c illustrates them for a single activity (A54); Figure 4b and 4d for the entire schedule. Note that all profiles decrease in a stepped manner. Its reason is the monotonicity of political apportionment methods, whereby “the number of seats accorded to any state [per that method does] not *decrease* if the house size *increases*” (Balinski and Young 1978, p. 280, emphasis in original). In other words, by adding one day to FLOAT, the method will assign it to one candidate activity, but keep all others’ constant. Note that LR does not exhibit monotonicity due to the Alabama paradox. Thus each candidate will occasionally receive a day from the gradually increasing FLOAT, which gives stepped shapes.

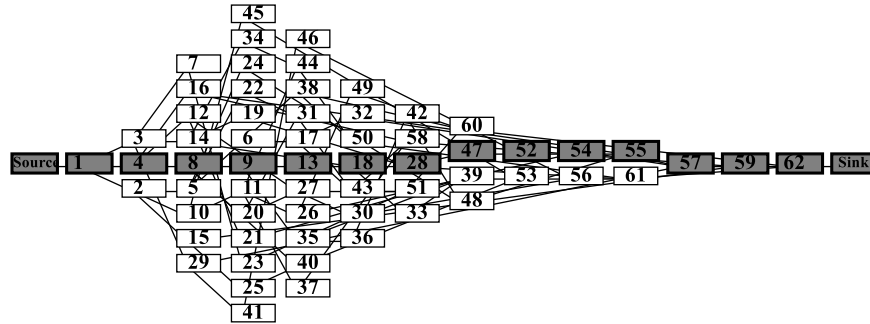
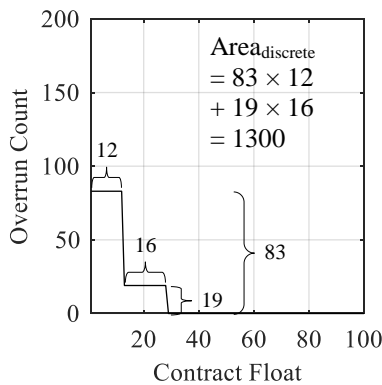


Figure 3: Network of J60 Schedule

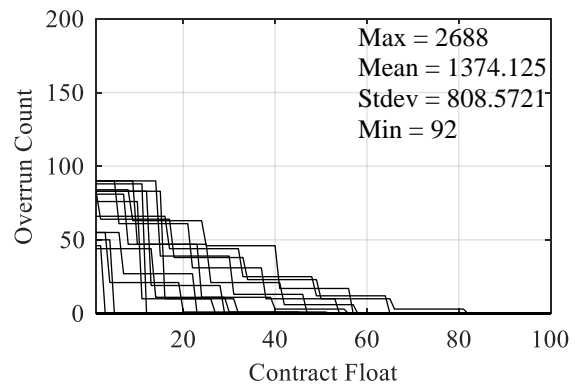
Since different combination of exponent n and apportionment method will affect the shape of performance profiles, a new way to identify trends is required. For this purpose the area performance index is developed. Per Equation 2, the area performance index for the discrete profile uses overrun counts at each FLOAT increment. Each contributes a vertical ‘stripe’ of unit width to the overall stepped profile. Adding these values from one to FLOAT gives the total area per Figure 4a. Mathematically speaking, this is equivalent to multiplying an overrun count with the range of FLOAT over which it applies, e.g. 83 instances of overrun across 12 days of FLOAT. An analogous calculation is performed for the continuous performance profile of Figure 4c. Note that saturating the entire schedule in Figures 4b and 4d requires 81 days of FLOAT. Clearly a smaller index value is better; it means that it has fewer overrun counts or shorter overrun periods and less required FLOAT to be saturated. This new metric fulfills Research Objective 1.

$$[2] \text{Area}_{discrete} = \sum_{i=1}^{FLOAT} \text{overrun_count}_i \times 1$$

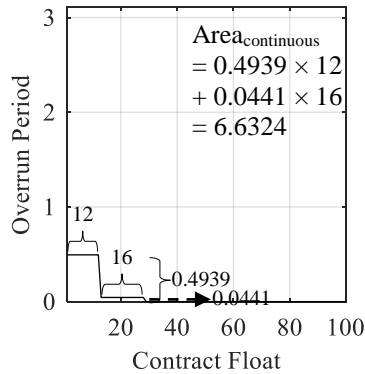
$$[3] \text{Area}_{continue} = \sum_{i=1}^{FLOAT} \text{overrun_period}_i \times 1$$



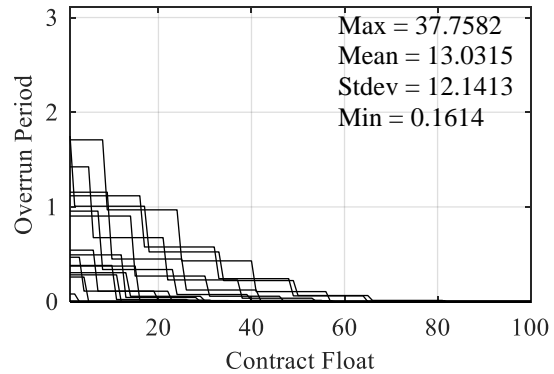
(a) Single Activity A54 (Discrete)



(b) Entire Schedule (Discrete)



(c) Single Activity A54 (Continuous)

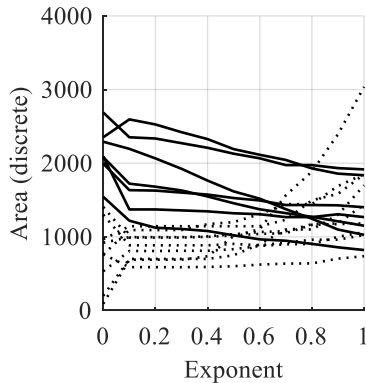


(d) Entire Schedule (Continuous)

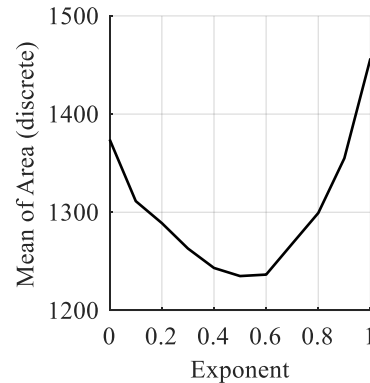
Figure 4: Performance Profile with Area Indices ($n = 0$, FLOAT = 100 and AM)

5 RESULTS

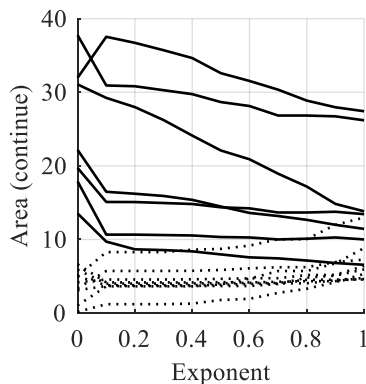
Gradually varying the exponent n from 0 to 1 and tracking the area performance indices for each activity, Figures 5a and 5c reveal trends in the discrete and continuous profiles. Note that if $n = 0$, every activity has a quota of $DRS^0 = 1$. Here the quota does not dominate to whom to allocate float, but the sequential order, i.e. a potentially unfair ‘first-come first-served’ policy applies. Thus results at $n = 0$ should be viewed separately from the remaining trend curve, which moves increasingly toward considering the quota more strongly as the exponent n becomes more dominant.



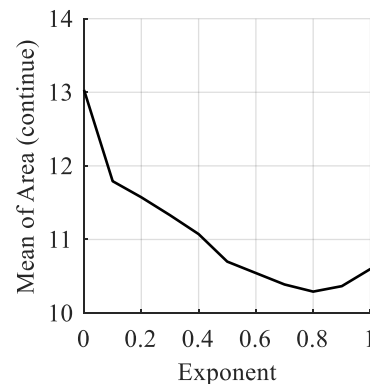
(a) Individual activity (Discrete)



(b) Entire Schedule (Discrete)



(c) Individual Activity (Continuous)



(d) Entire Schedule (Continuous)

Figure 5: Area Indices Profiles on Changing Exponent n from 0 to 1 (FLOAT = 100 and AM)

In Figures 5a and 5c, profiles with decreasing area indices are marked with solid lines, increasing ones with dotted lines. Interestingly, this categorization reveals that the former have quotas larger than the median and the latter have smaller ones: With growing n , the apportionment performance improves for larger quotas (solid) and worsens for smaller quotas (dotted). These two curve bundles cross one another. Theoretically, the average area indices for the entire schedule should reach a minimum value in the medium range of values for n . This is validated in the discrete and continuous profiles for the entire schedule of Figures 5b and 5d. Both profiles are concave, which means that their optimum exponent is between 0 and 1. Previous research by the authors has indicates that its value may also be influenced by the structure of the schedule as reflected by CI (Su *et al.* 2017b). Having generated and analyzed the performance profiles for candidate critical activities with both smaller and larger DRS fulfills Research Objective 2.

6 VALIDATION

Since LR suffers from the Alabama paradox, the other five apportionment methods (AM, HM, GM, GD, and SD) in Table 2 are considered for further simulation. Figures 4 and 5 have used AM for illustration purposes. Figures 6a and 6b now plot the relationship between the mean of discrete and continuous area indices and exponent to compare the different apportionment methods. These divisor methods AM, HM, GM, GD, and SD are magenta, cyan, green, blue, and dashed black. Figure 6 shows that discrete and continuous cases of GD have a minimum. Their optimum exponents are $n = 0.35$ and $n = 0.58$ for discrete and continuous cases. Having evaluated area performance indices to identify the best combination of apportionment method (i.e. rounding) and exponent fulfills Research Objective 3.

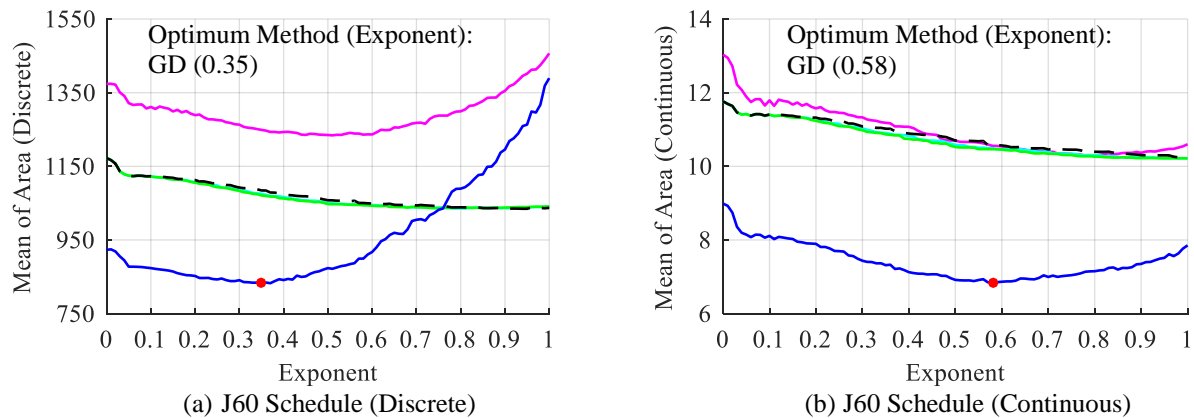


Figure 6: Mean of Area Performance Index Profiles over Exponents for Different Apportionment Methods

7 CONCLUSIONS

Carefully allocating float or buffers within a schedule can effectively protect it from delay. Expanding previous research that had proposed political apportionment methods for float allocation, this paper has illustrated how to optimally allocate it by developing new area indices, the first contribution to the body of knowledge. Graphically capturing the performance of these discrete and continuous measures for overrun count or period has explained their overall stepped shape. Trends within the area performance index for an individual activity have been explained, where it has been found that profiles for activities with a quota (DRSⁿ) larger than the median decreasing if the exponent n increases, and *vice versa*, the second contribution to the body of knowledge. The performance profiles for the entire schedule are concave as has been expected. Simulating different apportionment methods has allowed selecting the best method and its optimum exponent, the third contribution to the body of knowledge. This research has also determined when the important saturation with contract float occurs, which will best protect a given project.

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