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COMPLEX NETWORK ANALYSIS FOR WATER DISTRIBUTION SYSTEMS BY INCORPORATING THE RELIABILITY OF INDIVIDUAL PIPES

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Abstract: Complex network analysis (CNA) is commonly used to predict well-connectedness of the water main system using a deterministic technique. In this technique, pipeline network is idealized as nodes and links. A number of parameters are then defined based on the nodes and links to quantify the network connectivity. Pipeline system is, however, deteriorated with time due to various causes including corrosion, which may affect the connectivity of the network. The effect of pipeline deterioration on the network connectivity has not been well-investigated till to date. In this paper, water distribution network connectivity is evaluated using the CNA considering the pipeline deterioration. The effect of deterioration of individual pipes is expressed in terms of reliability of the pipe to incorporate in the CNA. The resulting time-dependent complex system parameters would be useful for maintenance planning of water main components. The proposed method has been demonstrated using a water distribution system of City of Mount Pearl in the province of Newfoundland and Labrador.

1. INTRODUCTION

Municipal water distribution systems play important roles in modern life through supplying potable water to the citizens. However, it has become a major challenge to the municipalities to maintain the integrity of these water systems. A number of breaks of water mains are observed every year across the municipalities around the world, associated with the deterioration of the pipes. While the breakage of one or more pipes in the water distribution network (WDN) may not affect the water availability to the communities due to the presence of redundancy in the network, the dysfunction of the pipes may overload the neighbouring pipes in the network. As are result, failure probability of the neighbouring pipes is increased. On the other hand, the breaks of some critical pipe components may isolate one or more communities from the network that may affect the water availability within those communities. The effects of water main breaks on the overloading of neighbouring pipes and isolation of networks should be properly considered for developing an effective maintenance program for municipal water infrastructure. It is however difficult to evaluate the effects for the complex WDN.

In the past, system reliability approach was employed for the assessment for the design of WDN with appropriate redundancy (Tung 1985). Wagner et al. (1988) developed analytical methods for system reliability assessment in terms of the connectivity and approachability. Quimpo and Shamsi (1991) employed the probability of the source node to communicate with demand node as the system reliability measure. Shinstine et al. (2002) used graph theory (e.g. minimum cut-set analysis) to obtain hydraulic system availability and reliability. The minimum cut-set analysis was also applied with the combination of fuzzy logic in a study of Yannopoulos and Spiliotis (2013). However, application of these methods for the assessment of complex WDNs consisting of thousands or more interconnected pipes would be very complicated.

Complex network analysis (CNA) has been proposed recently for quantification of structural properties of network and understanding the connectivity and robustness. Complex network models are being applied in various areas including WDNs (Yazdani and Jeffrey, 2010, 2011a, 2011b; Nazempour et al., 2016). Nazempour et al. (2016) used CNA for optimizing contamination sensor locations. Yazdani and Jeffrey (2010) introduce the method to evaluate robustness and vulnerable characteristic of WDN.

Three different types of measures are used in CNA. These are: (1) statistical measurements, (2) distance related measurements and (3) spectral-based measurements (Yazdani and Jeffrey, 2010, 2011a; Newman, 2010). The statistical measurements are mainly based on analysis of number of node and the link between the nodes. The metrics only provide a rough evaluation of network. In the distance related measurements, for un-weighted network, such as a social network, the length of path is calculated by number of nodes or links. For the weighted WDN, the length of path is obtained by the summation of distance from node to node within the path. Finding the shortest path is the fundamental algorithm, which is well known in the graph theory, for obtaining distance related measurements. In the spectral-based method, algebraic connectivity (AC) and spectral gap are employed. The AC is known as a measure for evaluating the well-connectedness of the graph (Fiedler, 1973; Capocci et al, 2005; Ghosh and Boyd, 2006; Newman, 2006, Yazdani and Jeffrey, 2011a, b). The network with higher AC is more robust or more tolerant to the breaks of links.

In the current paper, the spectral method with the AC of the CNA is applied in the assessment deteriorating WDN. The objective of the study is to investigate AC for the assessment of robustness and connectivity of WDN with application to a real municipal WDN at the City of Mount Pearl in the province of Newfoundland and Labrador. A method is proposed to determine to determine time-dependent AC to account for the deterioration of pipeline with time.

2. THE ALGEBRAIC CONNECTIVITY OF NETWORK

An algebraic connectivity (AC) is a parameter used in the graph theory to determine the strength of connection between the nodes in a network. It is defined as the second smallest Eigen-value of the Laplacian matrix of a connected graph (Fiedler, 1973). Municipal WDN can be considered as a graph of connected network where a number of pipes connect the nodes at the intersections. The graph of WDN could be described as G=G(V,E) where V is a set of n nodes (intersections) and E is a set of m pipes. An adjacency matrix A of G is used to describe the link between the nodes, where:

[1] $\mathrm{A} = \mathrm{a_{ij}}$

$\mathrm{a_{ij}} = 1$ if there is a link (pipe) between Node i and Node j.

$\mathrm{a_{ij}} = 0$ if there is no link (pipe) between Node i and Node j.

The node-degree matrix is a diagonal matrix which contains information about the number of connections (node degree) at each node, defined as:

$\mathrm{D} = \mathrm{diag}(\mathrm{d_i})$

$\mathrm{d_i}$: Number of connection (node degree) of Node i, where:

$\mathrm{d_i} = \sum_{j=1}^{n} \mathrm{a_{ij}}$

Then, the Laplacian matrix is given by (Eq.2):

[2] $\mathrm{L} = \mathrm{D} - \mathrm{A}$

The Laplacian matrix L for the undirected network is symmetric and the sum of rows (or columns) is zero. This characteristic leads to the fact that the first (the smallest) Eigen-value (λ_1) of the matrix is zero that corresponds to the Eigen vector of $(1,1,...,1)^T$. The second smallest Eigen-value (λ_2) of the Laplacian matrix is the AC (algebraic connectivity), which is greater than 0 if G is a connected graph. The Eigen-values of a network with n nodes of connected graph are: $\lambda_1 = 0 \le \lambda_2 \le \lambda_3 \le \cdots \le \lambda_n$.

The AC is widely used as a measure for the robustness of network and to quantify the number of pipe failures required to render a group of the nodes disconnected (Yazdani and Jeffrey 2011a). A network with higher AC implies that the network is better connected. Addition of number of links to a network was found to increase the AC (Gosh and Boyd 2006). On the other hand, if removal of a pipe or link leads to the disconnection of the network, the AC is increases for the graph with the same set of nodes (Fiedler1973). That is:

$$\lambda_2(G) \le \lambda_2(G_{re})$$

Where: $\lambda_2(G)$ is the AC of the connected network and $\lambda_2(G_{re})$ is the AC of the network after the disconnection with removal of edge(s).

The AC (or λ_2) for a few hypothetical networks shown in Figure 1 is calculated to demonstrate the numerical values of the parameter in describing robustness of the networks. Figures 1(a) to (h) show different scenarios of a network consisting of 10 nodes connected by 13 links between the nodes. The network is representative of a WDN connecting two small communities. One community contains nodes 1 to 5, and the other contains nodes 6 to 10. The communities are connected by a link between node 5 and node 6 (Figure 1(a)).

In Figure 1(a), all nodes in the network are connected with some degree of redundancies. All nodes are also connected in the networks in Figures 1(b) and 1(c). However, the redundancy is reduced in Figure 1(a) by removing the link between node 2 and node 5. The redundancy is further removed in Figure 1(c) by removing the link between node 2 and node 4. The AC for the three networks is calculated to be 0.1978, 0.1864 and 0.1830, respectively. The AC of the network is thus reduced with the reduction of the redundancy.

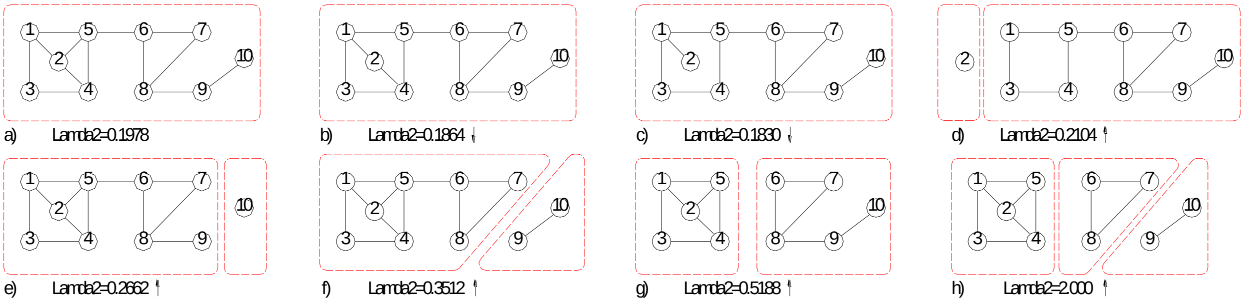

Figure 1. Examples of simple networks.

3. AC for Mount Pearl Water Network

In Figures 1(d) to 1(h), the networks are disconnected with removal of one or more links from the network in Figure 1(a). The AC in these cases is higher than those calculated for the connected networks (Figures. 1a to 1c), as expected (Fiedler 1973).A node or a network is isolated in Figures 1 (d) to 1(g), resulting into two networks. Two first Eigen-values are zero for these networks. However, the second smallest Eigen-values (the AC) are calculated to be 0.2104, 0.2662, 0.3512 and 0.5180, respectively, which are higher than the AC for the networks in Figures 1(a) to 1(c). It is to be noted that the AC is lower, when only a node is isolated (Figures 1d and 1e) while the parameter is higher when two networks of nodes are disconnected (Figure 1(g)). For isolation into three networks (Figure 1(h)), three Eigen-values are zero and the AC (the second smallest Eigen-value) is further increased to 2.000. Thus, a sudden increase of the AC with removal of link(s) provides an indication of disconnection of the network.

The complex network analysis of a medium size WDN of the City of Mount Pearl in the province of Newfoundland and Labrador is performed through calculation of the algebraic connectivity. The WDN at the city can be described using 4848 nodes connected by 5046 pipes (links). The network is shown in Figure 2. Some detail of the network is provided in Table 1. The WDN of the city of Mount Pearl includes several networks. Apparently, two main networks are connected by a pipe (Pipe no. 2489) as shown in Figure 3.

Figure 2. Water distribution network at the City of Mount Pearl

Table 1. Data of WDN at the city of Mount Pearl

Node ID	Pipe ID	Material	Diameter, mm	X	Y
1	1	Ductile Iron	150	320991.62230	5264475.70150
2	1	Ductile Iron	150	321007.09150	5264498.97460
3	2	Ductile Iron	200	321048.28280	5265709.24110
…	…	…	…	…	…
8564	9474	DI Private (approx.)	150	320423.51620	5263016.32110

Different parameters used in complex network analysis to assess the robustness and redundancy of the network are assessed for the WDN of the city. Yazdani and Jeffrey (2011a) provided a summary of the parameters used to assess redundancy and robustness of networks, as outlined in Table 2.The parameters calculated for the network at the city of Mount Pearl is provided in Table 3, which are compared with values for different other cities reported in Yazdani and Jeffrey (2011a). In Table 3, the parameters for the city of Mount Pearl are mostly within the ranges of those for the other cities, indicating that the redundancy, connectivity and robustness of the city network are similar to those for the cities.

The AC of 1.56e-5for the city of Mount Pearl is much less than those for the other cities. This may be due to the presence of high 'betweenness' nodes or pipes, such as pipe no. 2489 linking node 1738 to node 3518 of two main networks (Figure 3). The AC with removal of pipe 2489 is calculated to be 5.150e-5, a value close to the lower bound value for other cities in Table 3. Besides, the city network contains a significant number of one-degree nodes (i.e., 1156 out of 4848 nodes), representing the pipes leading to the consumption points, which may affect the AC to some extent.

Table 2. Complex system parameters to assess robustness and redundancy (modified after Yazdani and Jeffrey 2011a)

Types	Parameter		Equation
Statistical measurements	Total length	l_{total}	$l_{total} = \sum_{i=1}^{n} \sum_{j=1}^{n} l_{ij}$
	Number of pipe	m	-
	Number of node	n	-
	Link-per-node	e	$e = \frac{m}{n}$
	Average node degree	Aver_k	$\bar{k} = \frac{2m}{n}$
	Link density	q	$q = \frac{2m}{n(n-1)}$
	Independent loop	f	$f = m - n + 1$
	Mesh-ness coefficient	r_m	$r_m = \frac{f}{2n-5}$
	Threshold for random removal of node	f_c	$f_c = 1 - \frac{1}{\frac{\overline{k^2}}{\bar{k}} - 1}$
Distance related measurements	Route factor	g	$g = \frac{1}{1-n} \sum_{i=1}^{n-1} \frac{\varepsilon_{s,i}}{\delta_{s,i}}$
	Characteristic path length	l	$l = \frac{1}{n\left(1-n\right)} \sum_{i \neq j} d_{ij}$
Spectral measurements	Algebraic connectivity	λ_2	-
	Spectra gap	$\Delta\lambda$	-

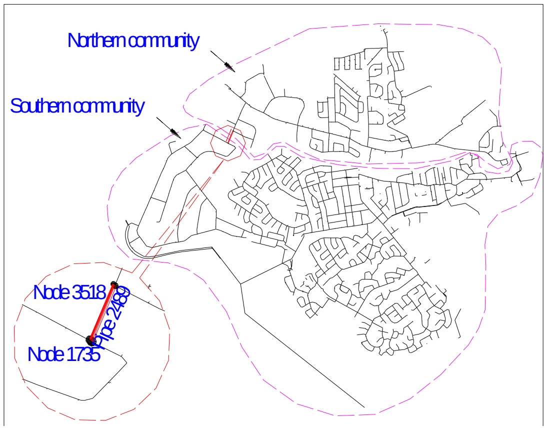

Figure 3.Connection of two main networks

Table 3. Complex system parameters for City of Mount Pearl network

Parameter		Value of MP city	Reference values*
Total length	l_{total}	129.837 (km)	
Number of pipe	m	5046	769-3065
Number of node	n	4848	755-2799
Link-per-node	e	1.041	1.01-1.10
Average node degree	Aver_k	2.08	2.04-2.23
Link density	q	4.29E-04	7.83e-4 - 2.7e-3
Independent loop	f	197	-
Mesh-ness coefficient	r_m	0.02032814	9.97e-3 - 5.86e-2
Threshold for random removal of node	f_c	0.18	0.22 - 0.42
Route factor	g	1.76	1.45 -1.67
Characteristic path length	l	88.38	25.94 - 51.44
Algebraic connectivity	λ_2	1.56e-05	6.09e-5 - 2.43e-4
Spectra gap	$\Delta\lambda$	2.55E-02	9.08e-3 - 7.27e-2

* From Yazdani and Jeffrey (2011a)

As above discussion, the AC can also be used to determine if break of any pipe in the network can result in the disconnection or isolation of the network. The City of Mount Pearl had 9 water main breaks in 2015. The city WDN has been investigated to identify if 9 water breaks can isolate the network. For this investigation, 9 pipes are selected randomly from the network. The AC of the network is then calculated. Figure 4 present the histogram of AC for 1000 dataset with 9 water main breaks randomly selected. In Figure 4, The AC is less than the AC of the intact network (i.e. 1.56e-5) for a number of cases (486 out of 1000 dataset). For these cases, the network connection will be retained, although the robustness of the network would be reduced.

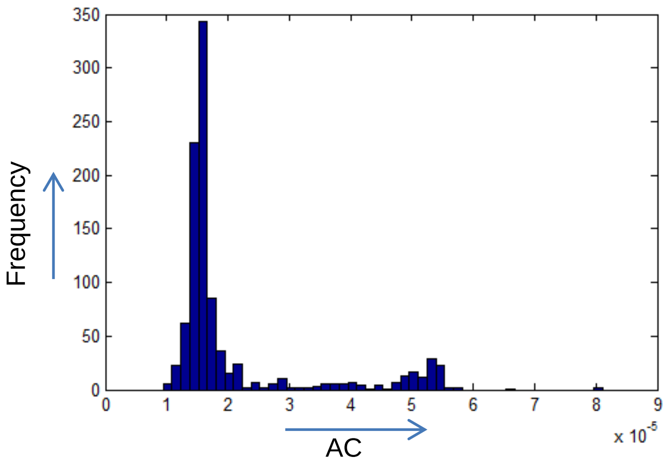

Figure 4. Histogram of Algebraic connectivity for 9 water main breaks

For the other cases (514 out of 1000 dataset), the AC is higher than the AC of the intact network, indicating that the breaks would lead to isolation (or disconnection) of the network. The values of AC from 1.56e-5 to 7.97e-5 imply different level of disconnection within the network from an isolation of several nodes to separation of network(s) to multiple pipes. For example, the highest AC in Figure 4 (7.97e-5) corresponds to the water main breaks locations shown in Figure 5. Figure 5 shows that breaking of the 9 water mains causes isolation of the network into 4 separate networks.

The study implies that the algebraic parameter can successfully be used to describe robustness and connectivity of WDNs.

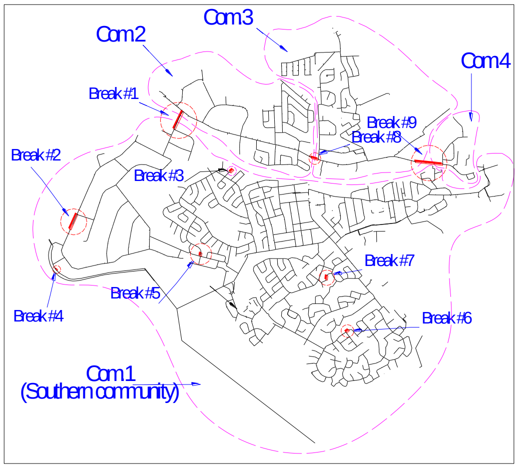

Figure 5. Locations of 9 water main breaks corresponding to AC = 7.97e-5

4. THE AC FOR DETERIORATING PIPES

The algebraic connection describe above is based on the adjacency matrix in Eq. 1. The adjacency matrix uses binary-based values of 1 or 0 to define if a link between two nodes exists or not. A value of 1 indicates that a link between two nodes exists. However, due to deterioration of pipes with time, the pipes may break, leading to the removal of the links between the nodes. To account for the time dependent deterioration, we proposed to define a time dependent adjacency matrix. The elements of the adjacency matrix provide a likelihood of having a link, defined as (Eq.3):

[3] $\mathrm{a_{ij}(t) = a_{ij}R_s(t)}$

Where,

$\mathrm{a_{ij}}$: Elements of the adjacency matrix defined in Eq. 1

$\mathrm{R_s(t)}$: Reliability of sth pipe connecting ith and jth nodes at time‘t’

Then the time depended node-degree matrix D(t) is defined as:

$\mathrm{D(t) = diag\big(d_i(t)\big) = \sum_{j=1}^{n} a_{ij}(t)}$

The time depended Laplacian matrix L(t) is obtained as (Eq. 4):

[4] $\mathrm{L(t) = D(t) - A(t)}$

The time dependent Laplacian matrix can be used to calculate the time dependent AC. The proposed method is applied to assess the WDN at the city of Mount Pearl. The reliability of the pipelines is determined based on water main break data, which is readily available to the city. For simplicity, the average breaks per year are taken as constant (Br). The event of water main break is considered to be a Poisson process. Poisson model is commonly used to forecast the break patterns in individual water mains (Kleiner et al., 2010).The average break rate for each pipe is assume to be proportional to the length of the pipe.

[5] $\mathrm{\lambda_s = \frac{Br}{l_{total}} l_s}$

Where, $\mathrm{\lambda_s}$: the average annual break rate of pipe s

Br: The average annual break rate in the WDN

$\mathrm{l_{total}}$: Total length of pipes in the network

$\mathrm{l_s}$: Length of pipes s

Then, the component reliability of pipe sth, which connects ith and jth nodes at time t is given by (Eq. 6):

[6] $\mathrm{R_s(t) = e^{-\lambda_s t}}$

Based on an annual rate of 9 breaks (Br = 9 break/year), encountered at the City of Mount Pearl, the Laplacian matrix for the city is calculated. Figure 6 shows how the Laplacian matrix changes with the incorporation of pipeline reliability. The figure shows the elements in the Laplacian matrix corresponding to nodes 2660, 2661, 2662 and 2664. These nodes are connected by pipes 348, 639 and 752.Locations of the nodes and the pipes in the network are shown in Figure 7.Reliabilities of these pipes at the age of 50 years are considered in Figure 6. Figure 8 shows the change of the AC of the network with time for applying the reliabilities of the pipes. Figure 8 shows almost a linear decrease of the AC from 1.562e-5 to 1.292e-5 from year 0 to year 50, respectively.

$$L = \begin{array}{c|ccccccccc|c}
\text{Node ID:} & 1 & \ldots & 2660 & 2661 & 2662 & 2663 & 2664 & \ldots & 4848 & \text{Node ID:} \\
\hline
 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
 & \ldots & \ldots & 2 & -1 & 0 & 0 & 0 & \ldots & \ldots & 2660 \\
 & \ldots & \ldots & -1 & 3 & -1 & 0 & -1 & \ldots & \ldots & 2661 \\
 & \ldots & \ldots & 0 & -1 & 2 & -1 & 0 & \ldots & \ldots & 2662 \\
 & \ldots & \ldots & 0 & 0 & -1 & 1 & 0 & \ldots & \ldots & 2663 \\
 & \ldots & \ldots & 0 & -1 & 0 & 0 & 2 & \ldots & \ldots & 2664 \\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 4848
\end{array}
\quad
L(t{=}50) = \begin{array}{c|ccccccccc|c}
\text{Node ID:} & 1 & \ldots & 2660 & 2661 & 2662 & 2663 & 2664 & \ldots & 4848 & \text{Node ID:} \\
\hline
 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
 & \ldots & \ldots & 0.747 & -0.061 & 0 & 0 & 0 & \ldots & \ldots & 2660 \\
 & \ldots & \ldots & -0.061 & 0.500 & -0.239 & 0 & -0.201 & \ldots & \ldots & 2661 \\
 & \ldots & \ldots & 0 & -0.239 & 0.240 & -0.001 & 0 & \ldots & \ldots & 2662 \\
 & \ldots & \ldots & 0 & 0 & -0.001 & 0.001 & 0 & \ldots & \ldots & 2663 \\
 & \ldots & \ldots & 0 & -0.201 & 0 & 0 & 0.590 & \ldots & \ldots & 2664 \\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 4848
\end{array}$$

Figure 6. The change of node 2661 and its links after 50 year in the Laplacian matrix

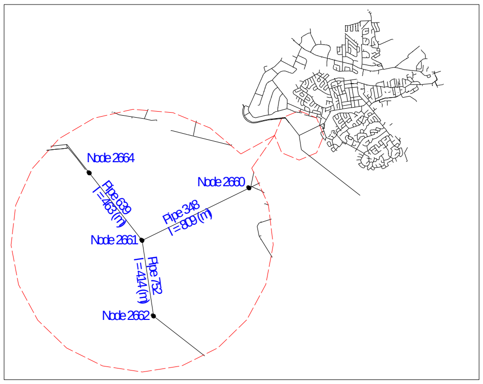

Figure 7.Locations of nodes 2660, 2661, 2662 and2664

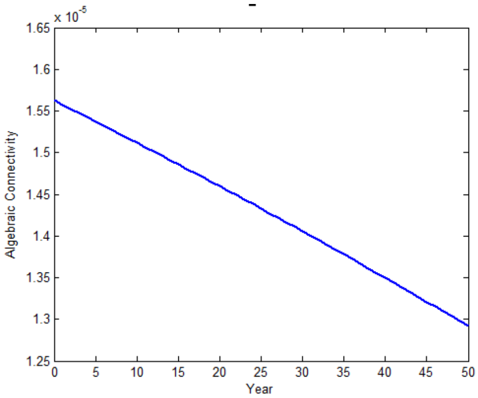

Figure 8. The change of AC with time

5. CONCLUSION

In this research, use of algebraic connectivity (AC) is investigated for the assessment of robustness of WDNs. The study reveals that the AC can effectively be used for the assessment of robustness and connectivity of municipal WDNs. In general, AC decreases with decrease of robustness or redundancy of the network. However, if breakage of pipelines leads to separation of the network, the AC would be suddenly increased. Comparison of the AC with the initial value of AC for the network could thus be used to evaluate the robustness and connectivity.

A method of calculating time-dependent AC is proposed to account for the deterioration of pipeline with age. Pipeline deterioration with time may reduce the degree of connectivity between the nodes. The reduction of degree of connectivity has been incorporated in terms of the reliability of individual pipes. For simplicity, the reliability is defined based on water main break record. The failure probability of the pipe is assumed to be proportional to the pipe lengths. The calculated time-dependent AC for the city of Mount Pearl decreases almost linearly with time for the assumed reliability of the pipe components. The dynamic AC can be compared with initial value of AC for the network and can be used for the maintenance planning of the WDN.

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