



## A PARAMETRIC STUDY ON TEMPERATURE AND SHRINKAGE CRACKING IN REINFORCED CONCRETE TANK WALLS

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**Abstract:** The design criteria in reinforced concrete tank walls subjected to temperature and shrinkage effects are important due to problems associated with cracking and leakage. There are major disagreement on the amount of minimum reinforcement proposed in current codes and standards for this purpose. The current design guides are based mainly on engineering judgment and past performance of these structures. In this study, the cracking behavior of reinforced concrete (RC) tank walls subjected to temperature and moisture variations is evaluated using the finite element (FE) method. Three-dimensional shell elements are used to discretize the models with steel reinforcement defined as smeared layers. The non-linearity of concrete is modelled using the brittle cracking constitutive model. The FE model is verified by comparing the crack width and crack pattern of a previous experimental study. The influence of tank wall dimensions, reinforcement ratio and climate is demonstrated on the width of developed cracks.

### 1 INTRODUCTION

Thermal and shrinkage cracks usually occur in concrete structures when a member is cast after the adjacent part is hardened. As fresh concrete undergoes volumetric deformations after casting, developing strains are restrained by adjacent parts that have hardened previously and this restraint is the reason for crack formation. Reinforcing the concrete with steel rebars is one of the common ways to reduce the crack widths in concrete structures. Wide-open cracks may occur in plain concrete and lead to its failure while in reinforced concrete rebars redistribute stresses after cracking resulting in several narrower cracks (Kianoush et al. 2008).

Cracking in reinforced concrete members has been studied by several researchers in the last few decades. Carlson and Reading (1988) investigated the stress conditions in base restrained walls having different shapes, considering various degrees of restraint by performing experiments on models made of rubber. ACI committee 224 (ACI 224.2R, 1992) has worked on crack control and prediction with more focus on flexural members and members under direct tension. ACI committee 207 (ACI 207.2R, 2007) discussed cracking due to restraint of thermal contraction in mass concrete. ACI 209 committee (ACI 209.2R, 2008) has provided guidelines for predicting shrinkage and temperature effects considering creep in concrete structures. Al Rawi and Kheder (1990), Kheder et al. (1994) and Kheder (1994) performed experimental studies on the behavior of base restrained RC walls. They suggested a crack prediction formula based on the theory of restraint variation during the formation of cracks. They concluded that the wall length to height ratio is one of the main factors influencing this type of cracking. Harrison (1981) studied early age thermal cracking with concentration on thermal contraction of concrete. He developed a theory to estimate the crack width based on bond force between the reinforcement and the adjacent concrete. Gilbert (1992) and Nejadi and Gilbert (2004) presented an approach to determine the spacing, number and width of cracks in fully restrained concrete members under direct tension using principles of mechanics. They concluded that cracking (crack spacing and width) in reinforced concrete members could be highly variable even in identical specimens. Pettersson and Thelandersson (2001, a & b) used the finite element program ANSYS to simulate base restrained walls. They defined non-linear springs to model the effect of reinforcements on crack widening. The tensile strength of the concrete has the most significant influence on the crack width according to their results. Thelandersson et al. (1998) studied the behavior of end restrained walls due to sudden and slow changes in temperature using a computer program. They simulated temperature in concrete and came to conclusion that rapid temperature change in walls is less critical in terms of cracking in comparison to slow temperature changes. Therefore, a lower percentage of reinforcement is required to control the crack width in walls under rapid temperature change mainly because of the formation of internal restraint. Elbadry and Ghali (1995) investigated thermal cracking in pre-stressed concrete and concluded that partial pre-stressing



could be more effective than full pre-stressing since it allows the concrete tensile stresses to be relieved by crack formation.

Existing guidelines on the required reinforcement to control the width of cracks proposed by design codes are very limited and generic. In fact, practical difficulties and significant costs of full scale experiments have limited the number of relevant research studies in this field. Therefore, the relation between crack width and reinforcement ratio or other factors that affect the crack width is still not clearly established. This study is performed to clarify the influence of parameters that could influence the crack width in RC tanks.

## 2 Cracking behavior of reinforced concrete

Since restrained volumetric deformations due to shrinkage and temperature drop usually leads to development of tensile strains, the possibility of cracking in the concrete is high. The tensile stress caused by shrinkage and temperature drop increases with time to a point that it reaches the tensile strength of concrete leading to crack formation. Thermal and shrinkage induced tensile strains can reach 1000 micro-strains, however, concrete may be able to resist not much more than 100 micro-strains (Kianoush et al 2008). In reinforced members, after the first crack is formed, the stresses will be transferred to the reinforcement from the concrete. Therefore, after the crack is formed, there will be zero stress in the concrete at the location of crack while tensile stresses start to develop in the steel. By further increasing the stresses, more cracks start to develop and therefore the stresses get redistributed after the formation of each crack. This procedure will last until the tensile stresses become stable in the member.

### 2.1 FE Material Modeling

Brittle cracking model has been chosen for simulating the nonlinear behavior of concrete since it best describes members mainly under tension in which the compressive failure is of no significance. This model assumes a linear elastic behavior for concrete in compression and considers anisotropy that could be caused by crack development. A simple brittle failure criterion is used to allow the removal of elements from the mesh. ABAQUS uses the Rankine criterion for detecting the initiation of crack. Based on this criterion, as soon as the maximum principle stress reaches the tensile strength of concrete, a crack starts to develop. When a crack forms it remains until the end of calculation therefore it is not reversible. Nevertheless, it might open or close along the directions of the crack surface normal. In addition to tension stiffening, post-cracked behavior has been defined to consider shear retention of reinforced concrete. This behavior is based on the fact that after crack opening, the shear modulus at cracked surface reduces. Therefore shear stiffness could be defined based on the opening strain across the crack. In case of reinforced concrete, after the element failure occurs based on brittle cracking criterion, the element stress carrying capacity reduces to zero but the rebars start to contribute to carry stresses. By including a shear failure criterion in the rebar material, the progressive failure of under-reinforced concrete member could be modeled where concrete failures is followed by a ductile failure of reinforcement.

### 2.2 Crack width calculation

Since ABAQUS does not offer an option to determine the crack width directly, a formulation proposed by Frosch (1999) is used to calculate the width of cracks. Frosch proposes that the crack width has a direct relationship with steel strain and crack spacing as presented in the equation below:

$$W_c = \varepsilon_c S_c \quad \text{Eq 1}$$

Where,  $W_c$  is width of crack,  $\varepsilon_c$  is the strain in reinforcement and  $S_c$  is the spacing between cracks. This equation is also adapted in ACI 318-02 code for calculating the crack width. Based on statistical data, Frosch also developed an equation for spacing between cracks ( $S_c$ ) as presented below:

$$S_c = \psi_s d^* \quad \text{Eq 2}$$



Where  $\psi_s$  is defined as crack spacing factor (1 for minimum crack spacing, 1.5 for average crack spacing and 2 for maximum crack spacing) and  $d^*$  is called controlling cover distance. Frosch calibrated the crack spacing factor value based on cracking in flexural members in his model. Here in this study, assuming that the crack width due to volume change is still proportionate to steel strain at the cracked location, this factor is adjusted based on the experimental results on RC walls. Controlling cover distance ( $d^*$ ) could be calculated using the following equation having  $S$  as the reinforcement spacing and  $d_c$  as the distance from the concrete surface to the centre of the first layer of reinforcements.

$$d^* = \sqrt{d_c^2 + \left(\frac{S}{2}\right)^2} \quad \text{Eq 3}$$

### 3 Case study and FE verification

In order to verify the selected finite element model, the crack pattern and width of a simulated wall is compared with the ones of an experimental study performed by Kheder (1997). The selected wall is 3.25 m tall and 12 m long with a thickness of 500 mm and 0.2% reinforcement ratio. The volumetric strain was measured to be 465  $\mu\epsilon$ . The crack width, spacing, height and pattern were recorded using monitoring devices. The bar diameter is 12 mm with a spacing of 262 mm for 0.2% reinforcement ratio. The rebars are simulated by defining the bar section area and spacing between them. The reinforcements are modeled as smeared layers with a thickness equal to area of a bar divided by the bar spacing. The reinforcement cross section area is evenly distributed among the concrete elements. Therefore, the bar spacing and size do not directly influence the results and only the reinforcement ratio will be of importance in the model. The calculated temperature and shrinkage strain in the case study is modeled by applying the equivalent temperature change to the walls using the following equation considering the coefficient of thermal expansion ( $\alpha$ ) of  $10 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$ .

$$\Delta T = \frac{\epsilon}{\alpha} \quad \text{Eq 4}$$

Therefore the equivalent temperature change of 46.5  $^\circ\text{C}$  is applied to the simulated walls. The material properties as modeled in the FE model are presented in table 1. The concrete is modeled to behave linearly until it reaches its tensile strength at 3 MPa with a linear post-failure tension stiffening behavior. The mesh is selected to be 100x100 mm after performing the mesh sensitivity analysis on 3D shell elements. The wall was allowed to dry for a total of 6 months excluding a 7 day moist curing period. Creep is considered in the model using a concept known as compliance which is defined as the total load induced strain (the summation of elastic and creep strain) at age  $t$  per unit stress due to sustained loading applied since the age of  $t_0$ . ACI 209R-92 model for calculating creep and shrinkage suggests using the following equation for compliance:

$$J(t, t_0) = \frac{1 + \phi(t, t_0)}{E_{cm(t_0)}} \quad \text{Eq 5}$$

In which,  $E_{cm(t_0)}$  is the elastic modulus at the time of loading and  $\phi(t, t_0)$  is the coefficient of creep defined as the ratio of creep strain to the elastic strain at the start of loading at age  $t_0$  days. The temperature change is also applied in a time-dependent manner using the following equation assuming the ultimate strain is equal to the measured 465  $\mu\epsilon$  since it was reported that the samples did not experience that much strain after the specified period.

$$\epsilon_{sh}(t, t_c) = \frac{(t - t_c)^\alpha}{f + (t - t_c)^\alpha} \epsilon_{shu} \quad \text{Eq 6}$$

The strain contour of the FE model for the selected wall is presented in Figure 1 along with the cracking pattern observed in the case study for the same wall. The widths of cracks are calculated based on the steel strain using Frosch equation. A crack spacing factor of  $\psi = 1$  is considered based on the crack spacing measured in the experiments. Not only the cracks widths are in the same range but also the overall crack pattern of FE study is in good agreement with the case study. Therefore, based on this comparison, the proposed FE model seems to be in good agreement with the experiments and the selected constitutive model is proven to be capable of simulating the RC walls behavior under volumetric changes.

Table 1. Material properties of concrete and steel

Material property	Concrete	Steel
Elastic modulus (MPa)	25000	200000
Tensile Strength (MPa)	3	--
Yield Strength (MPa)	--	400
Density (Kg/m <sup>3</sup> )	2400	7800
Poisson's ratio	0.18	0.3
$\alpha$ (°C)	10-5	--

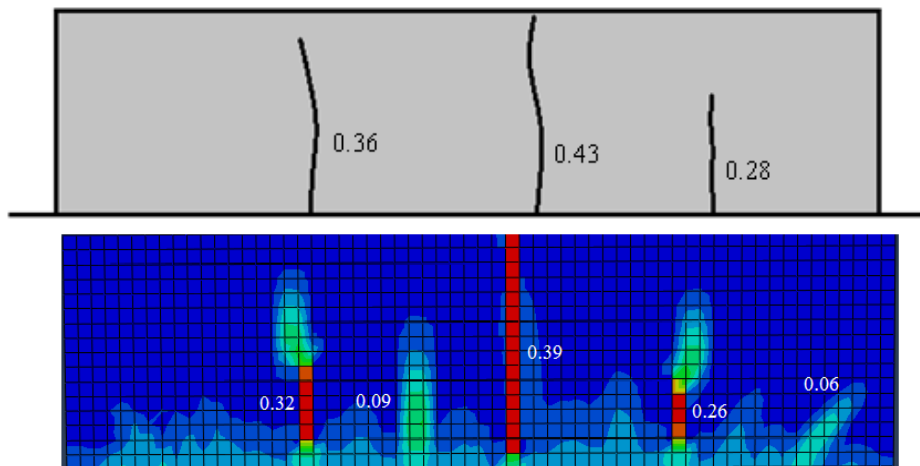


Figure 1. Crack pattern and width, experimental study versus FE model ( $\rho=0.2\%$ ,  $d=12$  mm,  $H=3.25$  m,  $L=12$  m,  $e_v=465 \times 10^{-6}$ )

#### 4 Parametric study

ACI 350 recommends maintaining the crack width below 0.27 mm and 0.23 mm for flexural members under normal and severe environmental conditions respectively. ACI 224R however, proposes the allowable crack width for structures based on the exposure conditions. According to this committee report the allowable crack width for water tight structures should be limited to 0.1 mm.

The influence of length, height, length/height ratio, thickness, reinforcement ratio and volumetric strain is discussed on the crack width of RC tanks. The default height, thickness and volumetric strain were assumed to be 4 m, 500 mm and 600  $\mu\epsilon$  respectively. The default concrete tensile strength was assumed to be 3 MPa and the reinforcement ratio was taken as 0.3% unless otherwise stated.

Table 4. Variables considered for the parametric study

Parameter	Value
Length (L)	4, 6, 8, 10, 12, 15, 20, 30 (m)
Height (H)	4, 6, 8 (m)
Thickness (t)	300, 500, 700 (mm)
Volumetric strain ( $\epsilon$ )	400, 600 and 800 ( $\mu\epsilon$ )
Reinforcement ratio ( $\rho$ )	0.2, 0.3, 0.4, 0.5, 0.6, 0.7 (%)
Concrete tensile strength (ft)	2.6, 3, 3.4 (MPa)

##### 4.1 Temperature and shrinkage strain



A reasonable value for shrinkage and temperature strain is needed to be considered for the parametric study since it significantly affects the crack width of structures subjected to volumetric deformations. Wider cracks may develop when the structure is exposed to higher temperature and shrinkage strains. In order to determine the total thermal strain, the maximum and minimum concrete temperature need to be estimated. Having the total temperature change, the thermal strain could be calculated by multiplying the coefficient of thermal expansion by this magnitude. The hydration temperature is not considered as the highest temperature since the concrete undergoes compressive stresses as the temperature increases due to hydration. Therefore, the maximum temperature could be taken as the ambient temperature during concrete casting. In case of liquid containing structures, the temperature of water rarely goes below 0 °C and that prevents the concrete temperature to drop below 0 °C in the winter. Therefore, considering a maximum temperature of 30 and a minimum of 0 °C the total temperature change could be taken as 30 °C which is equivalent to a volumetric strain of 300 µε assuming the thermal coefficient expansion ( $\alpha$ ) to be  $10 \times 10^{-6}$  °C.

The shrinkage strain on the other hand, is calculated using the procedure proposed by ACI 209 (ACI 209.2R-08). ACI 209 committee proposes the shrinkage strain to be calculated using Equation 6 to be equal to 300 µε. A total volumetric strain of 600 µε is considered for the RC tanks, which is the summation of shrinkage and temperature strain.

#### 4.2 Finite element model

A non-linear tension stiffening behavior for concrete and an elastic-plastic constitutive model for steel is defined in the FE program. The steel reinforcement is defined to behave perfectly plastic after yielding. The reinforcement is provided in two layers in horizontal and vertical directions with a clear cover of 50 mm. The spacing between the bars was defined to be 200 mm. The tank walls are completely fixed to their base with zero degree of freedom and tied to each other assuming no relative movement at their intersections. The total volumetric strain which is the summation of shrinkage and temperature strain is applied to the tanks in form of a temperature reduction. This temperature drop was taken to be 60 °C corresponding to a volumetric strain of 600 µε assuming a coefficient of thermal expansion ( $\alpha$ ) of  $10 \times 10^{-6}$  °C. After performing the analyses, the magnitude of steel strain is extracted from the results to calculate the width of cracks using Frosch equation. The controlling cover distance ( $d^*$ ) is calculated based on reinforcement spacing ( $S$ ) of 200 mm and clear cover ( $d_c$ ) of 50 mm as follows:

$$d^* = \sqrt{d_c^2 + \left(\frac{S}{2}\right)^2} = \sqrt{50^2 + \left(\frac{200}{2}\right)^2} = 112 \text{ mm}$$

Therefore, crack spacing can be found using the following equation by having the crack spacing factor of 1 and controlling cover distance of 112 mm:

$$S_c = \psi_s d^* = 112 \text{ mm}$$

Consequently, the crack width can be calculated as:

$$W_c = S_c \varepsilon_s = 112 \varepsilon_s \text{ mm}$$

Therefore, the crack width of the RC tanks is computed using the abovementioned equation by having the maximum steel strain magnitude from the FE analyses.

### 5 Results and Analyses

The influence of the tank walls' length, height, thickness and length/height (L/H) ratio in addition to applied volumetric strain is investigated on the width of cracks by conducting a series of FE analyses. In order to study the influence of length of tank walls on crack widths, tanks are modeled with lengths from 4 to 30 m and heights of 4, 6 and 8 m considering thickness of 500 mm. A volumetric strain of 600 µε is applied to all 24 simulated tanks with constant reinforcement ratio of 0.3%. Figure 2 presents the results of the FE analyses demonstrating that tanks with longer walls experience wider cracks. The lowest crack width which is about 0.1 mm occurs in the 4x4 m tank and 4x30 m tank seems to experience the widest crack of about 0.8 mm.

Figure 2 presents the influence of L/H ratio on the width of cracks. It varies between 1 and 3.5 considering 3 different heights. For L/H = 3.5 for example, the length of the tank walls are 14, 21 and 28



m for 4, 6 and 8 m tall walls respectively. It is shown in this figure that tanks with higher L/H ratio will experience wider cracks. Considering 4 m tall tanks as an example, the crack width varies from 0.08 mm to 0.49 mm as the L/H ratio increases from 1 to 3.5.

Effect of tank height on the crack width is shown in Figure 9. The crack width is compared with three different heights (4, 6 and 8) while the L/H ratio varies between 1 and 3.5 and the rest of parameters are kept constant. It can be concluded from the graph that in tanks having the same L/H ratio, wider cracks develop in tanks with taller walls.

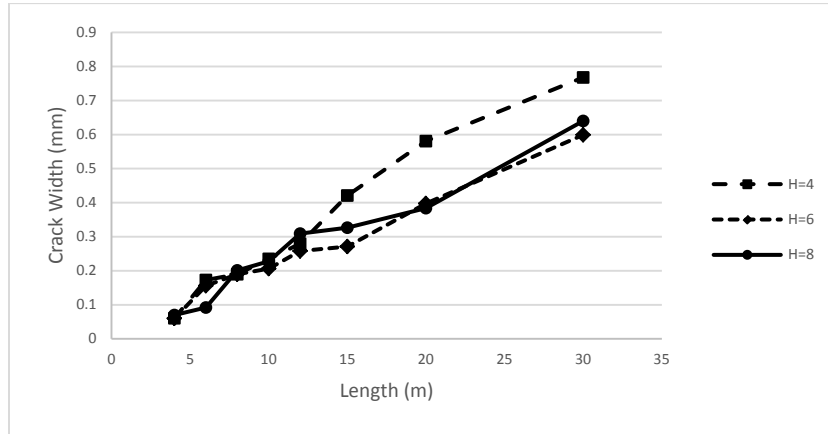


Figure 2. Effect of length on crack width for different heights

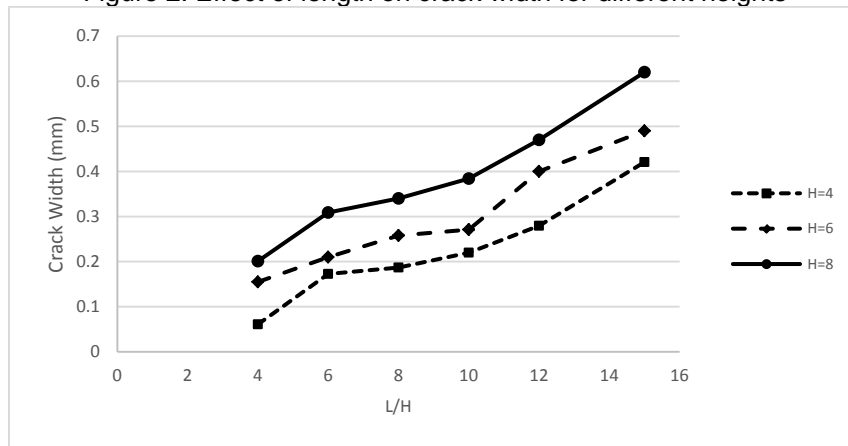


Figure 3. Effect of L/H on crack width for different heights

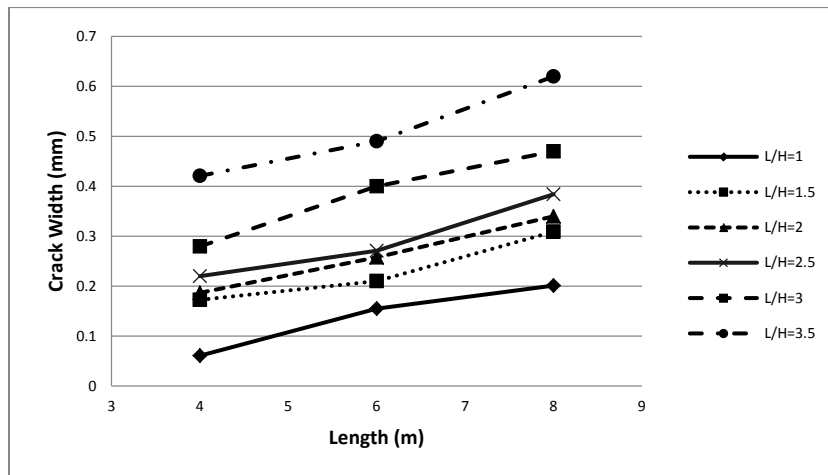


Figure 4. Height versus crack width considering constant L/H ratios



Figure 5 presents the crack width versus length for tanks having three different thicknesses (300, 500 and 700 mm). It should be noted that volumetric strain is different for each wall thickness. Therefore, the strain is calculated for 300 and 700 mm thick walls using Equation 6. The calculated shrinkage strain for 300 and 700 mm thick tanks are 361 and 256  $\mu\epsilon$  respectively. It is noticeable that thinner walled tanks experience higher shrinkage strain which is because of their higher exposed surface to volume ratio. As per the graphs presented in Figure 5, thickness is of no significant influence on the crack width since there seem to be very little to no difference in crack width of the simulated tanks with different thicknesses.

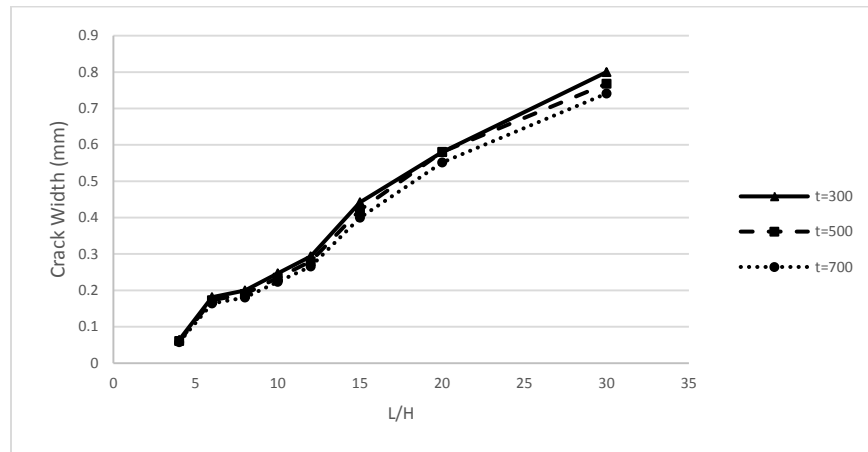


Figure 5. Effect of wall thickness on crack width

The amount of applied volumetric strain which is the summation of strain due to moisture loss and temperature reduction could also affect the crack width significantly. This magnitude highly depends on the climate of the area in which the tank is being cast. Assuming a dry and hot weather for example, the temperature variation could be up to more than 40 °C (maximum of 40 °C and minimum of 0 °C) which corresponds to 400  $\mu\epsilon$ . The shrinkage strain on the other hand can also be calculated using the method presented earlier. Assuming a relative humidity of 20% for the wall thickness of 500 mm, the shrinkage strain can be calculated as 500  $\mu\epsilon$ . Therefore, the total volumetric strain for such climate could be 900  $\mu\epsilon$ . Here in this parametric study, two extreme climates (hot/dry and cold/humid) are assumed and the volumetric strains are applied to the RC tanks based on the calculations while the rest of parameters are kept constant. In addition, a volumetric strain of 300  $\mu\epsilon$  is also added to them for the sake of comparison. Figure 6 illustrates the results of different volumetric strains applied to the tanks. It can be seen that in hot/dry weather (volumetric strain = 900  $\mu\epsilon$ ) the crack width is higher than cold/humid weather condition (volumetric strain = 600  $\mu\epsilon$ ). The graphs show that for tanks having walls with lengths varying from 4 to 30 m, the crack width varies from 0.1 mm to 1 mm in hot/dry climate. Whereas crack width of the same tanks constructed in cold/humid climate vary between 0.1 mm and 0.8 mm.

Effect of reinforcement ratio on the width of cracks is demonstrated in Figure 7. Tanks are simulated with reinforcement ratios varying from 0.3% up to 0.7% with their wall lengths changing between 4 and 30 m. The results show that for tanks with the lowest reinforcement ratio (0.3%), the crack width goes up to about 0.8 mm for the longest wall (30 m long). Whereas for the same tank size, reinforcing the tank with 0.5% or 0.6% steel reduces the crack width to less than 0.3 and 0.2 mm. Based on this figure, it could be concluded that since the crack width for water retaining structures should be kept below 0.1 mm, the reinforcement ratio of 0.5% is enough for tanks which are less than 6 m long but it should be increased to 0.6% or even 0.7% for the tanks with longer walls.

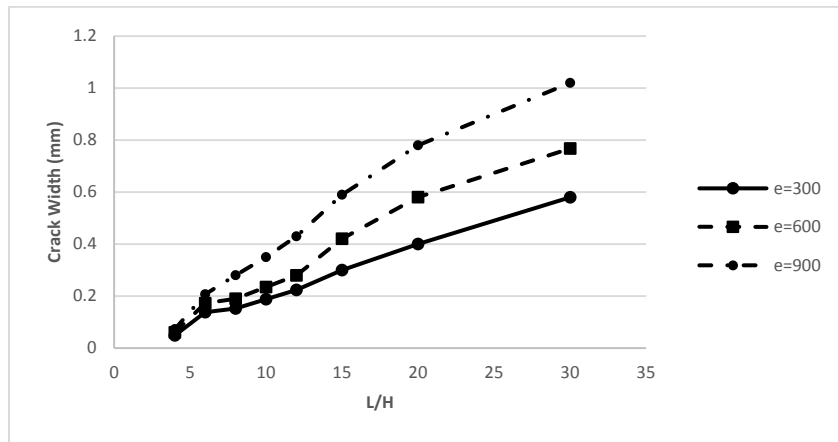


Figure 6. Effect of volumetric strain on crack width

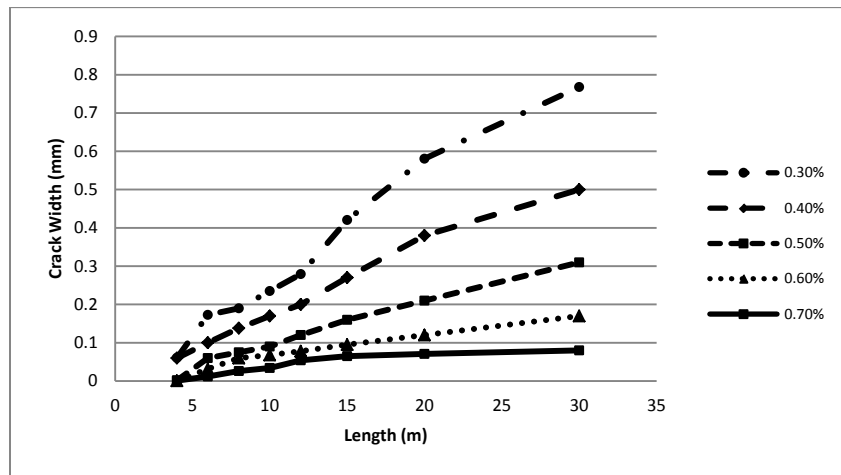


Figure 7. Effect of reinforcement ratio on crack width

## 6 Conclusion

The cracking behavior of RC tanks subjected to shrinkage and thermal strain was studied using finite element method. The selected FE model was first verified by comparing the crack width and pattern of the model with an experimental study. A parametric study was conducted in order to investigate the effect of a series of parameters on crack width. The effect of tank walls' length, height, width and L/H ratio, reinforcement ratio and the influence of climate on the width of cracks. It was shown that tanks with longer walls and higher L/H ratio experience wider cracks and considering a constant L/H ratio, the crack width is larger in tanks with taller walls. It was also shown that the wall thickness does not have a significant effect on the crack width considering three different thicknesses. The climate that the tanks are being cast in was demonstrated to be of importance for this type of cracks. In fact, wider cracks develop in hot and dry climate as opposed to cold and humid environment. Increasing the reinforcement ratio was proven to reduce the width of cracks considerably. RC tanks with reinforcement ratio from 0.3% up to 0.7% were simulated and exposed to thermal and shrinkage strain and the results showed that 0.7% of steel could reduce the crack width to an acceptable level for tanks with long walls that experience the widest cracks.





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