INVESTIGATION OF THE BEHAVIOUR OF PIPE ELBOWS DUE TO THE BOURDON EFFECT FOR DIFFERENT BOUNDARY CONDITIONS

Md Abdul Hamid Mirdad, Graduate Student, Structural Engineering, University of Alberta, Canada
Diana Abdulhameed, Graduate Student, Structural Engineering, University of Alberta, Canada
Muntaseer Kainat, Ph.D., Structural Engineering, University of Alberta, Canada
Marwan El-Rich, Assistant Professor, Structural Engineering, University of Alberta, Canada
Michael Martens,Sr. Mechanical/Pipe Stress Engineer, TransCanada Pipelines Ltd, Canada
Samer Adeeb, Associate Professor, Structural Engineering, University of Alberta, Canada
J. J. Roger Cheng, Professor, Structural Engineering, University of Alberta, Canada

Abstract: Steel pipe elbow is a common feature in piping systems which is used to change the direction of a straight pipe. The Bourdon effect is the phenomenon of the elbows tending to straighten out when pressurized. Shemirani et al. (2014) studied the influence of the Bourdon effect on the ovalization developed in the elbows for fixed boundary conditions. They showed that there is an increase in the stresses within the elbow; however, this effect is not at all addressed in the Canadian oil and gas pipeline code CSA Z662-11. Kainat et al. (2014) studied how geometric imperfections influence the Bourdon effect in the elbows for fixed boundary conditions. Here we extended the study of the Bourdon effect by focusing on the behavior of pipe elbows using different boundary conditions and neglecting any initial geometric imperfections. Nominal pipe sizes of 12, 36 and 42 inch were analyzed with various OD/t ratios. 90º pipe elbow models were analyzed using finite element analysis software ABAQUS for different boundary conditions at the ends of the elbows. The ends have been allowed to expand in the radial direction upon pressurization to prevent stress concentrations. Under normal operating pressure for (80% SMYS) hoop stress; the Bourdon effect, von Mises stresses and reaction force in the pipe elbows were investigated. From the analysis results, the effect of the boundary conditions on the mechanical response of the elbow is investigated.

1. INTRODUCTION

Elbows, cold bends and induction bends are used to change the direction of a pipeline. The surface geometric characteristics of pipe elbowsmay lead to a host of unbalanced thrust forces caused by the internal pressure and temperature. The internal pressure could affect the pipe elbow in two ways; first, the “stiffening” of the pipe elbow as the internal pressure increase. The stiffening results from the internal pressure resisting the ovalization of the cross section that normally occurs when a bending moment is applied to the bend or when the bend starts to straighten out. The internal pressure causes unbalanced thrust forces tending to open up the elbow fitting or bend depending on its stiffness and surrounding constraints. These unbalanced thrust forces have a notable effect on the flexibility which can cause stresses and deformations that may not be accounted for in traditional stress analysis (Gaurav and Girish, 2013; Hong et al., 2011). These additional forces tending to cause ovalization of the cross section and causing the tendency of pipe bends to open up are termed the "Bourdon effect".

When the pressurized pipe elbow tends to straighten out and the cross section starts to ovalize, the outer fiber gets closer to the neutral axis leading to a decrease in the moment of inertia and the section modulus, and causesan increase in the pipe stresses. The Bourdon effect is an important issue that needs to be considered in the design of elbows and it is not included in the current Canadian oil and gas pipeline code (CSA Standard Z662-11, 2011).Our previous study investigated the effect of internal pressure on the behavior of the pipe elbow by evaluating the displacement at the center of the mid section of the pipewhich was considered to be representing the Bourdon effect (Shemirani et al., 2014). That study showed that in case of elbows with no initial ovality,the displacement of the center increases by increasing the pipe diameter (Shemirani et al., 2014). However, thicker elbows have higher stiffness and lower displacement values than thinner pipe elbows.

The purpose of this study is to investigate the additional forces and stresses acting on the elbows due to its geometry. In this paper, the behavior ofpipe elbows with different end constraints subjected to internal pressure was analyzed and presented. Two cases of end constraints were considered in this study, which represent two extreme scenarios. A fixed-fixed constraint was considered as the lower bound and the fixed-free constraint was considered as the upper bound. A mathematical calculation is presented in this paper to evaluate the additional thrust forces depending on the internal pressure and pipe elbow radius.

2. INVESTIGATION OF THE PIPE ELBOW BEHAVIOUR USING FINITE ELEMENT ANALYSIS

A finite element analysis (FEA) is conducted for pipe elbows with nominal sizes of 12, 36 and 42 inches. Models of 90º pipe elbowswerecreated and analyzed using the FEA ABAQUS software for the two cases of end constraints. Under normal operating pressure corresponding to 80% SMYS (Specified Minimum Yield Strength) hoop stress, the Bourdon effect, von Mises stresses and reaction force in the pipe elbows were investigated separately.

2.1 Geometry and Properties of Pipeline Elbow

Outer diameter to wall thickness ratio (OD/t) of 40, 70 and 100 were considered. According to the Barlow formula, the internal pressure corresponding to 80% SMYS in the hoop direction is constant for the same OD/t ratio. Table 1 shows the geometric parameters of analyzed pipe elbows.

Table 1: Geometric parameters of pipe elbow

NPS	Outside Diameter, OD (mm)	Radius of Curvature of Elbow (mm)	Elbow Thickness, t (mm)	OD/t	Pressure, P for 80% SMYS (MPa)
12	323.85	466.73	8.10	40	14.40
			4.63	70	8.23
			3.24	100	5.76
36	914.40	1352.55	22.86	40	14.40
			13.10	70	8.23
			9.14	100	5.76
42	1066.80	1581.15	26.67	40	14.40
			15.24	70	8.23
			10.67	100	5.76

The ratio of the radius of curvature of the elbow andthemean cross sectional radius of the pipe was taken as the standard value of 3 for a long radius pipe (Weldbend, 2010). The pipe elbows were discretized using S4R shell elementswhichare 4 node elements with 6 degrees of freedom in each node. The steel was considered as an isotropic linear elastic plastic material with Young's modulus of 207 GPa and Poisson's ratio of 0.3. X52 grade steel which is widely used in pipeline with yield stress of 360 MPaandultimate stress of 460 MPa (API 5L X52, 2012) was considered. Nonlinear geometric analysis option was used for analysis. Pipe elbows were chosen to be open at the top and bottom in all cases for fluid flow. Figure 1 shows the typical geometry and the finite element mesh of one of these models.

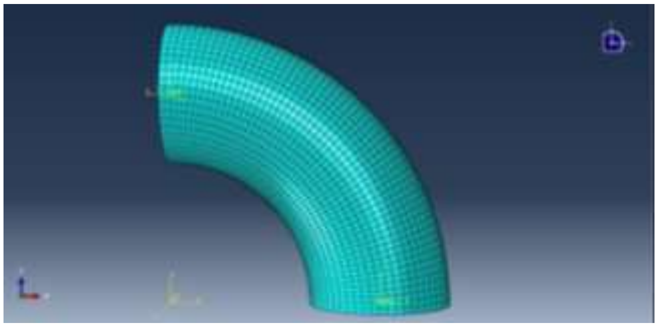

Figure 1: The typical geometry and mesh of the ABAQUS 3D model of pipe elbow

90º pipe elbowswere analyzed in this study for 2 different cases of boundary conditions at the bottom and top of elbows which are-

Case-1: Fixed at the bottom and top of pipe elbow.
Case-2: Fixed at the bottom and free at the top of pipe elbow.

In Case-2, out of plane translation and rotation were constrained such that the curved axis of the elbow remains on the X-Z plane during deformation. The boundary conditions at the support of the pipe elbowswere applied to a reference point which was created at the centre of the pipe elbow at the support. Then, with respect to this reference point a kinematic coupling constraint was applied, which allows the pipe to expand in radial direction. This modelling technique prevented stress concentrations at the supports upon pressurization.

2.2. Evaluating the Thrust Forces on a Pipe Elbow

An infinitesimal area on the pipe surface defined and enclosed by the angles dφ and dψ where,dφ is the in-plane angle for the pipe elbow, while dψ is the infinitesimal angle along the circumference of the pipe elbow as shown in Figure 2.

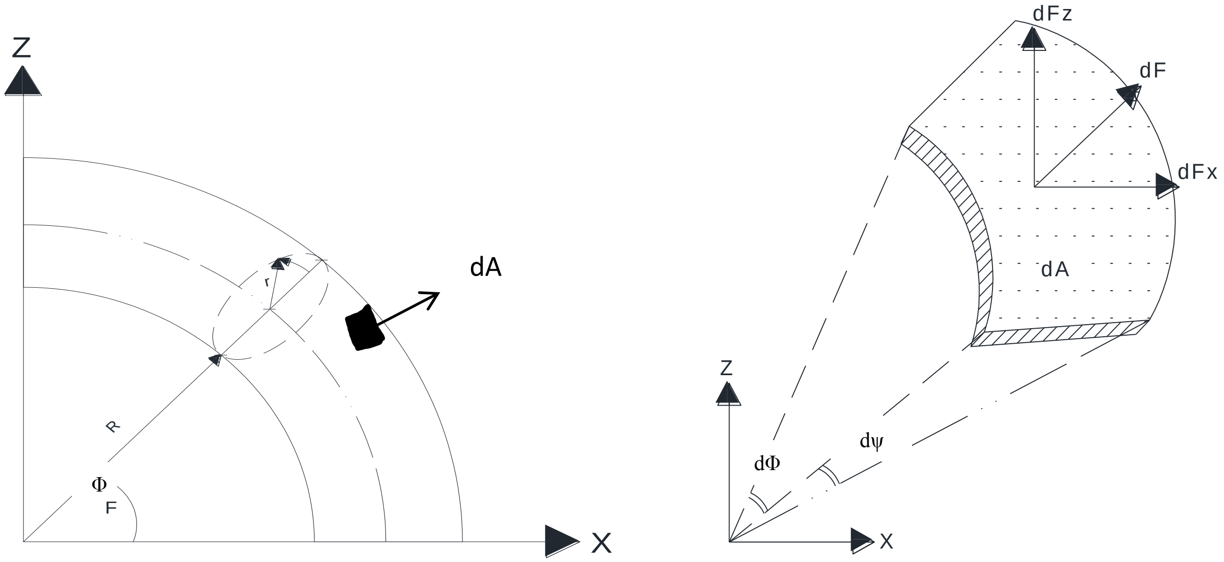

Figure 2: Pipe elbow profile and geometry.

The thrust forces (dF) acting on the infinitesimal area under the effect of the internal pressure "P" is calculated as follows:

$\mathrm{dA} = (\mathrm{R} + \mathrm{r}\cos\psi)\mathrm{d\Phi}.\mathrm{r}\,\mathrm{d\psi}$
$\mathrm{dF} = \mathrm{P}.\mathrm{dA}$
$\mathrm{dF} = (\mathrm{PRr} + \mathrm{Pr}^2\cos\psi)\mathrm{d\Phi}.\mathrm{d\psi}$
$\mathrm{dF_x} = \mathrm{dF}.\cos\psi.\cos\Phi$
$\mathrm{dF_z} = \mathrm{dF}.\cos\psi.\sin\Phi$

Where, R is the inside radius of the elbow, r is the radius of the pipe and P is the internal pressure. The total force acting on the pipe elbow surface can be evaluated by integrating the area with respect to ψ and Φover an angle of 2π and π/2 respectively. The additional total thrust forces in X and Z directions acting on the elbow are as follows;

$$[1]\mathrm{F_x} = \int_0^{\pi/2}\int_0^{2\pi} (\mathrm{PRr} + \mathrm{pr}^2\cos\psi)\cos\psi.\cos\Phi\,\mathrm{d\psi}.\mathrm{d\Phi} = \mathrm{Pr}^2\pi$$

Similarly,

$$[2]\mathrm{F_z} = \mathrm{Pr}^2\pi$$

These thrust forces calculated from Equations 1 and 2 are applicable to both fixed-fixed and fixed-free boundary conditions.

3. RESULT AND DISCUSSION

3.1 von Mises Stress and Bourdon Effect

The Bourdon effect is the phenomenon of the elbows tending to straighten out when pressurized. The Bourdon effect causes one end of the elbow to displace and/or rotate relative to the other end of the elbow. In the current study, for the fixed-fixed case the displacement at the middle section of elbow is considered as the measurement representative of the Bourdon effect while in the fixed-free case the displacement at the top of the elbow in the horizontal (X) and vertical (Z) directions is considered. This displacement measurement provides a method for comparing the pipe elbow systems when the geometric properties of this system change.

Table 2 shows the maximumvon Mises stress and the Bourdon effect displacement for the fixed-fixed boundary condition after the application of internal pressure corresponding to 80% SMYS hoop stress.

Table 2 : Maximum von Mises stress and Bourdon effect displacement for the fixed-fixed boundary condition

NPS	OD/t	Pressure, P for 80% SMYS (MPa)	Maximum von Mises Stress (MPa)	Displacement (mm)	Displacement (% OD)
12	40	14.40	305.4	0.41	0.13%
	70	8.23	298.8	0.43	0.13%
	100	5.76	299.2	0.44	0.14%
36	40	14.40	312.7	1.23	0.14%
	70	8.23	306.4	1.29	0.14%
	100	5.76	307.0	1.31	0.14%
42	40	14.40	313.7	1.43	0.13%
	70	8.23	307.2	1.50	0.14%
	100	5.76	306.9	1.53	0.14%

For the fixed-fixed boundary conditions, the OD/t ratio has insignificant (around 5% variation) effects on the displacement magnitude for all three elbow sizes. The displacement magnitudes are seen to increase with increasing the elbow size. However, when the magnitudes are expressed as %OD, it is observed that the displacements are essentially similar across different elbow sizes. For the same applied internal pressure in all nominal pipe sizes, the maximum von Mises stress increases with increasing the nominal pipe size but decreases with increasing OD/t ratio. The maximum von Mises stress in the fixed-fixed case is well below the yield stress of 360 MPa with an average factor of safety of 0.85.

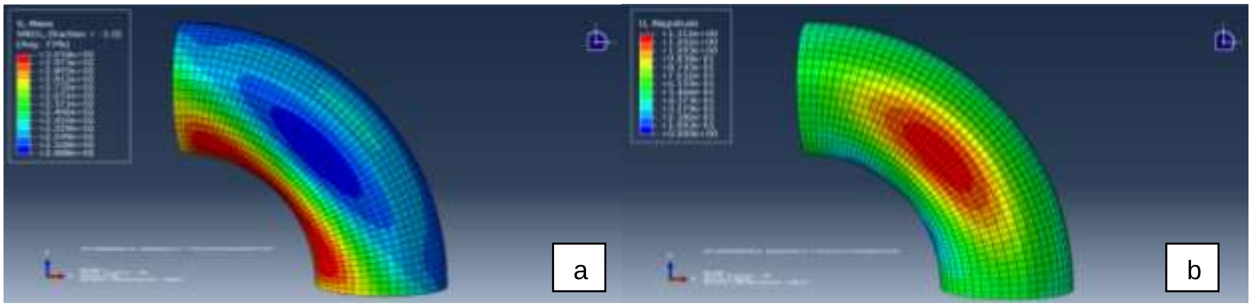

Figure 3: For the fixed-fixed elbow a) von Mises stress of a NPS 12 model and b) Displacement pattern of a NPS 36 model

For the fixed-fixed boundary condition, the maximum von Mises stress is at the intrados of the elbows while the maximum displacement is at the middle of elbows (Figure 3).

Table 3 : Actual pressure, pressure at failure and modified elbow thickness for the fixed-free boundary condition

NPS	OD/t	Elbow Thickness t (mm)	Pressure, P for 80% SMYS in Pipe (MPa)	P_f Pressure at Failure [exceeding yield & ultimate stress] (MPa)	$P_{80\%}$ Actual Pressure at Stress of 80% SMYS in Elbow (MPa)	Modified Elbow Thickness to sustain 80% SMYS pressure, t (mm)
12	40	8.10	14.40	9.06	3.02	38.55
	70	4.63	8.23	4.85	1.73	22.03
	100	3.24	5.76	3.15	1.21	15.42
36	40	22.86	14.40	8.81	2.88	114.3
	70	13.10	8.23	4.71	1.65	65.33
	100	9.14	5.76	3.03	1.15	45.72
42	40	26.67	14.40	8.87	3.02	127.00
	70	15.24	8.23	4.77	1.73	72.58
	100	10.67	5.76	3.10	1.21	50.80

In the fixed-free boundary condition, the elbow thickness is not sufficient enough to withstand the internal pressure corresponding to 80% SMYS of hoop stress in a straight pipe. In this case, the thickness of the elbow needs to increase to be able to sustain the same pressure corresponding to 80% SMYS for a straight pipe. Table 3 shows the actual pressure causing 80% SMYS in the elbow ($P_{80\%}$) which is the pressure at which the von Mises stresses in the elbow reach 80% SMYS (around 288MPa). Table 3 also shows the modified thickness of elbows that would allow the elbow to sustain the same 80% SMYS pressure as a straight pipe. In all cases, $P_{80\%}$ is equal to 20% of the pressure causing 80% SMYS in a straight pipe. Pressure at failure P_f is also found from the finite element analysis when the von Mises stresses in the elbow reaches the ultimate tensile strength (460 MPa). Similarly, the modified elbow thickness is found to be equal to 1/20% (five times larger) of the thickness of a straight pipe such that both would have the same pressure causing 80% SMYS. Table 4 and 5 show the maximum von Mises stress and the Bourdon effect displacement for the fixed-free boundary condition considering P_f and $P_{80\%}$.

Table 4 : The maximum von Mises stress and the Bourdon effect displacement for the fixed-free boundary condition at P_f

NPS	OD/t	P_f Pressure at Failure [exceeding yield & ultimate stress] (MPa)	von Mises Stress (MPa)	Displacement		Displacement	
				Horizontal (mm)	Vertical (mm)	Horizontal (%OD)	Vertical (%OD)
12	40	9.06	460	15.20	14.70	4.69%	4.54%
	70	4.85	460	12.58	12.45	3.88%	3.84%
	100	3.15	460	8.13	8.51	2.51%	2.63%
36	40	8.81	460	43.81	42.44	4.79%	4.64%
	70	4.71	460	34.83	34.67	3.81%	3.79%
	100	3.03	460	22.21	23.45	2.43%	2.56%
42	40	8.87	460	53.82	51.92	5.04%	4.87%
	70	4.77	460	51.03	49.49	4.78%	4.64%
	100	3.10	460	28.57	29.79	2.68%	2.79%

Table 5 : The maximum von Mises stress and the Bourdon effectdisplacement for the fixed-free boundary condition at $P_{80\%}$

NPS	OD/t	$P_{80\%}$ Actual Pressure at Stress of 80% SMYS in Elbow (MPa)	von Mises Stress (MPa)	Displacement		Displacement	
				Horizontal (mm)	Vertical (mm)	Horizontal (%OD)	Vertical (%OD)
12	40	3.02	291.4	1.11	1.39	0.34%	0.43%
	70	1.73	292.4	1.16	1.48	0.36%	0.46%
	100	1.21	292.0	1.18	1.51	0.36%	0.47%
36	40	2.88	284.8	3.17	3.97	0.35%	0.43%
	70	1.65	285.0	3.30	4.20	0.36%	0.46%
	100	1.15	284.3	3.35	4.28	0.37%	0.47%
42	40	3.02	291.1	3.89	4.87	0.36%	0.46%
	70	1.73	294.5	4.10	5.15	0.38%	0.48%
	100	1.21	294.9	4.12	5.26	0.39%	0.49%

The horizontal & vertical displacement magnitudes increase with increasing the elbow size and increasing OD/t ratios for $P_{80\%}$ but for P_f the results are reversed. For P_f, the OD/t ratio has very significant (around 90% variation) effects on the displacement magnitude but for $P_{80\%}$ the OD/t ratio is very insignificant (around 5%) effect for all three elbow sizes. When the magnitudes are expressed as %OD, it is observed that the displacements are essentially similar across different elbow sizes for $P_{80\%}$ while completely different at P_f.

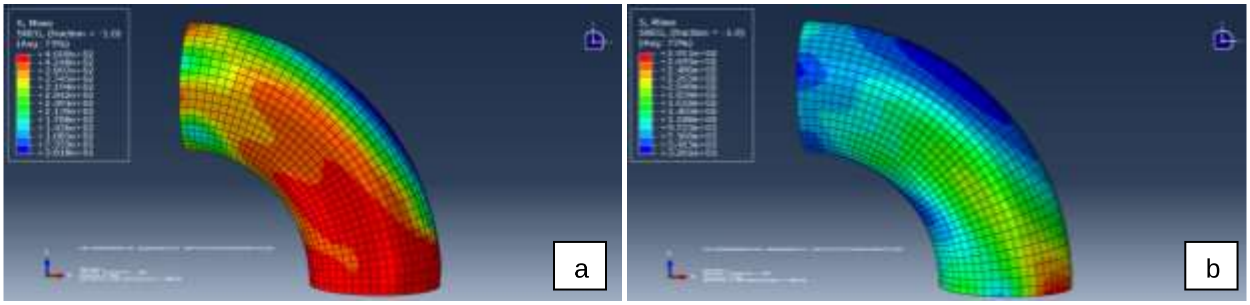

Figure 4: For the fixed-free elbow of NPS 42 model a) von-Mises stress at failure pressure, P_f and b) von Mises stress at $P_{80\%}$

In Figure 4(a) we can see that, for the fixed-free boundary condition, the maximum von Mises stress is at the bottom support region while in the fixed-fixed boundary condition it was at the intrados of the elbow. The elbow fails around the bottom support region at P_f (Figure 4(b)).

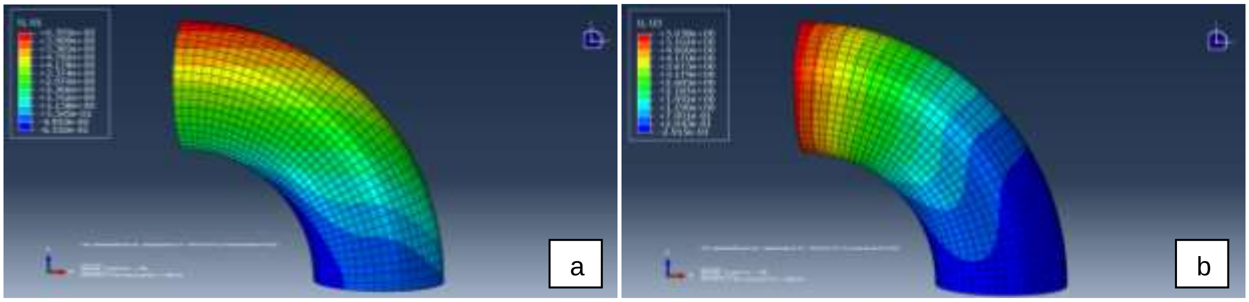

Figure 5: For the fixed-free elbow of NPS 42 model a) Horizontal displacement pattern and b) Vertical displacement pattern

The fixed-fixed and fixed-free boundary condition at the end of elbows discussed in this study can be considered as the extreme cases. The behaviour of pipe elbow is somewhere in between thes two cases. Internal pressure corresponding to 80% SMYS can be sustained by fixed-fixed elbows similar to straight pipe with a slight increase in the stress (85% SMYS). But for the fixed-free case only 20% of applied internal pressure can be sustained by the elbows due to the Bourdon effect. For the extreme case of fixed-free boundary condition, the elbow thickness needs to increase 5 times to be able to bring the stresses down to the 80% SMYS level. It is customary to increase the thickness of the elbow by 1.5 to 2 times, however, our study shows that the boundary conditions of the elbow which is governed by the flexibility of the attached pipes are very important elements for identifying the required increase in the elbow thickness.

3.2 Thrust Forces

In this section, the results from the mathematical Equation 1 and 2 for the thrust forces is compared to the results from the FEA. The thrust forces in the FEA were evaluated from the end reaction forces at both ends of the pipe elbow. For the fixed-fixed pipe elbow, the thrust forces were equal to the summation of the reactions from both ends. However, For the fixed-free end pipe elbow, the thrust forces were equal to the end reaction at the fixed end.Comparison between the FEA and the equation is shown in the Tables 6 and 7 below-

Table 6 : Thrust force for the fixed-fixedboundary condition

Fixed- Fixed						
NPS	OD / t	Applied Pressure (MPa)	FEA (KN)		Equation [F= $Pr^2\pi$] (KN)	
			RF 1	RF 2	RF 1	RF 2
12	40	14.40	1094.58	1094.58	1094.32	1094.32
	70	8.23	625.59	625.59	625.43	625.43
	100	5.76	437.84	437.84	437.72	437.72
36	40	14.40	9190.87	9190.87	9190.86	9190.86
	70	8.23	5252.85	5252.85	5252.83	5252.83
	100	5.76	3676.40	3676.40	3676.35	3676.35
42	40	14.40	12561.79	12561.79	12560.18	12560.18
	70	8.23	7179.49	7179.49	7178.49	7178.49
	100	5.76	5024.79	5024.79	5024.07	5024.07

Table 7 : Thrust force for the fixed-free boundary condition

NPS	OD / t	Applied Pressure (MPa)	Pressure at Failure [exceeding yield & ultimate stress] (MPa)	Fixed-Free			
				FEA (KN)		Equation [$F= Pr^2\pi$] (KN)	
				RF 1	RF 2	RF 1	RF 2
12	40	14.40	9.06	686.88	676.76	688.33	688.33
	70	8.23	4.85	367.93	361.56	368.38	368.38
	100	5.76	3.15	238.65	234.46	239.00	239.00
36	40	14.40	8.81	5610.86	5518.59	5624.81	5624.81
	70	8.23	4.71	3003.71	2950.96	3008.28	3008.28
	100	5.76	3.03	1932.13	1894.82	1934.85	1934.85
42	40	14.40	8.87	7721.00	7585.70	7733.30	7733.30
	70	8.23	4.77	4157.76	4074.18	4163.52	4163.52
	100	5.76	3.10	2700.07	2642.64	2703.49	2703.49

From Table 6 and 7 for the fixed-fixed and fixed-free boundary conditions, the reactions obtained from the FEA model are the same as the reactions evaluated from the proposed mathematical equations. The FEA and mathematical equation results (Table 6 & 7) show that increasing the OD/t ratio will result in a decrease in the reaction forces,while increasing the pipe nominal size will cause an increase in the reaction forces. Interestingly, the proposed equation for the thrust forces show that they are only a function of the applied internal pressure and the pipe radius.

4. CONCLUSIONS

4.1 von Mises Stress and Bourdon Effect

The von Mises stress was used as a measure of the stress level in the analyzed elbows. The results of the analysis show that the OD/t ratio does not have a significant effect on the maximum von Mises stress in both case studies of the elbow. In the fixed-fixed case, the maximum stress reached levels slightly higher than 80% of SMYS, however, for the fixed-free case, the increase in the stress was much higher and to bring the stresses back to 80% of SMYS a 5 times increase in the wall thickness is required.

The measurment representitive of the Bourdon effect in case of fixed-fixed pipe elbows were considerd to be the displacement at the centre of the middle section. While,in the case of fixed-free pipe elbows, the horizontal and vertical displacements at the top of the free endwere considered. The results of analyzed elbows show that the larger the elbow diameter the larger the displacement in both cases. In terms of the elbow thickness, thicker elbows have higher stiffness and experience less displacement than thinner ones. For both boundary condition cases, The OD/t ratio was found to be effective on the displacement values at the failure pressure by about 90% however at 80% pressure the effect of OD/t on displacement was insignificant. When the magnitudes are expressed as %OD, it is observed that the displacements are essentially similar across different elbow sizes for both case studies. So modification of the elbow thickness might be a suitable option.

4.2 Thrust Forces

The mathematical equation presented in this paper to evaluate the additional thrust forces in a pipe elbow was found to be function of the internal pressure and the pipe radius. The results from the proposed equations were almost the same reactions calculated from the finite element analysis for both case studies. The thrust forces in the finite element analysis were evaluated from the end reaction forces at both ends of the pipe elbow. For pipes having same diameter and OD/t ratio,the thrust forcesevaluated for the fixed-fixed case show higher values than in the fixed-free case. However, for pipes having the same boundary conditions and same OD/t ratio, it was found that the reaction forces increase with the increase of the pipe size as predicted by the equation.

REFERENCES.

[1] Gaurav, B. and Girish, T. May-June. 2013. Stress Intensification & Flexibility in Pipe Stress Analysis.*International Journal of Modern Engineering Research*, Vol.3, Issue.3, pp-1324-1329.

[2] Hong, S.P., An, J.H., Kim, Y.J., Nikbin, K. and Budden. P. May 2011. Approximate Elastic Stress Estimates forElbows Under Internal Pressure.*International Journal of Mechanics Sciences*, 53 (2011) 526-535.

[3] Canadian Standards Association (CSA).June 2011. *Oil and Gas Pipeline System*.CSA Standard Z662-11, Mississauga, ON, Canada.

[4] Shemirani, F., Adeeb, S., Cheng, J.J.R. and Martens, M. 2014. Investigation of the Bourdon Effect Influence on the Stress and Ovalization in Elbows, *10*th International Pipeline Conference, IPC 2014, Calgary, Alberta, Canada.

[5] Kainat, M., Cakiroglu, C., Adeeb, S., Cheng, J.J.R. and Martens, M. 2014. Geometric Analysis of Pipeline ElbowsThrough Reverse Engineering and Their Associated Imperfections, *10*th International Pipeline Conference, IPC 2014, Calgary, Alberta, Canada.

[6] Welbend Corporation 2010. *Carbon Steel Weld Fitting & Weld Flange Products for Piping Construction.* 63rd edition. Welbend Corporation, Argo, Illinois, USA.

[7] American Petrolium Institute 2012. *API Specification 5L for Line Pipe.*45th edition. American Petrolium Institute, Washington, D.C., USA.