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Functionally Graded Piezoelectric Sphere in Constant Magnetic Field Under Hygrothermal Loading

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Abstract: In this paper, a closed-form solution is obtained for hygrothermal stresses in a functionally graded piezoelectric (FGP) sphere placed in an external constant magnetic field. The sphere could also be rested on a Winkler-type elastic foundation. The power-law formulation with different non-homogeneity indices is adopted for the variation of material properties along the thickness of the sphere. The moisture concentration and temperature distributions through the thickness of the sphere are found by solving the steady-state Fickian moisture diffusion and Fourier heat conduction equations. The two coupled governing differential equations including the Lorentz force effect are solved analytically. Analytical expressions are obtained for displacement, electric potential, radial stress, and hoop stress distributions in the FGP sphere. Finally, the numerical results are illustrated to reveal the effect of moisture concentration and temperature changes, external magnetic field, elastic foundation, different non-homogeneity indices, and aspect ratio on the multiphysical behavior of the FGP sphere. The analytical results could be significant for accurate and reliable design of adaptive wood structures and could be used as a benchmark for validating the numerical methods employed in the multiphysical analysis of smart structures.

1 Introduction

Piezoelectric materials exhibit interesting multiphysical phenomena; the electric field is generated when they are mechanically deformed, and vice versa. Similar interaction could be found in piezoelectric materials among electric, temperature, and moisture fields because of the pyroelectric and hygroelectric effects [Patron and Kudryavtsev, 1988]. The study of physical interaction among elastic, electric, thermal, and hygroscopic fields is could be called as hygrothermoelectroelastic analysis [Altay and Dokmeci, 2007]. Regarding the multifunctional ability of piezoelectric materials in sensing, actuating, and load carrying and considering their applications in various external and environmental conditions, a better understanding on the multiphysical interaction of smart structures composed of piezoelectric components is crucial for the development of adaptive structure industries.

On the other hand, conventional laminated composites, with the likelihood of failure and delamination at the interface of different layers, are gradually being replaced by functionally graded materials (FGMs) with continuous transition of material properties. Although FGMs were first introduced as thermal barriers to sustain high temperature gradient, they have lots of potential applications in microelectromechanical system, automobile industry, and biomedical devices [Reddy and Chin, 1998, Akbarzadeh et al., 2011]. Furthermore, regarding the application of FGMs and hygrothermoelectroelastic media in the present of magnetic field, the Lorentz force could affect the structural behavior [Akbarzadeh et al., 2011]. Consequently, the magneto-hygrothermoelectroelastic analysis of functionally graded (FG) structures is significant for accurate design of FG smart structures.

Smittakorn and Heyliger [2000] studied the coupled hygrothermopiezoelectric behavior of laminated plates using the discrete-layer method. Ootao and Tanigawa [2005] investigated multi-layered magnetoelectrothermoelastic strips under transient and non-uniform thermal loadings employing the hybrid Laplace-Fourier transform. Altay and Dokmeci [2008] acquired a four-field variational principle for the motion and deformation of a hygrothermopiezoelectric medium. A closed-form solution was obtained by Liu [2011] for the bending of magnetoelectroelastic rectangular thin plates under different mechanical boundary conditions and based on the classical plate theory. Recently, Akbarzadeh and Chen [2012] investigated the uncoupled hygrothermomagnetoelectroelastic behavior of an infinitely-long hollow rotating cylinder rested on Winkler-type elastic foundation using an analytical approach.

A theoretical method was given by Wang and Dong [2006] for analyzing the magnetothermoelastic responses in an FG conducting cylinder using the finite Hankle integral transform. The generalized magneto-thermoelasticity in an FG one-dimensional viscoelastic layer was studied by Ezzat and Atef [2011]. Akbarzadeh et al. [2012] obtained closed-form solutions for the classical coupled thermoelasticity analysis of FG rectangular plates based on the third-order shear deformation theory. An analytical solution was presented by Dai et al. [2012] for the time-dependent behavior of a hollow FGP sphere in constant magnetic field under thermal loading. The theoretical analysis of an FG hollow thermomagnetoelectroelastic cylinder was conducted by Ootao and Ishihara [2012]. In addition, the hygrothermoelectroelastic responses of one-dimensional FG media working in a constant magnetic field were studied by Akbarzadeh and Chen [2013].

Accordingly, this paper investigate the multiphysical behavior of a Functionally graded piezoelectric (FGP) sphere under combined loadings of mechanical, electrical, temperature, and moisture fields in the presence of a constant magnetic field. The closed-form solutions are obtained for the uncoupled magneto-hygrothermoelectroelasticity which can be significant for the design of smart structures, with potential application in civil engineering for energy harvesting and health monitoring, and could be used as a benchmark for validation of numerical methods employed for solving the multiphysical problems.

2 Problem Definition and Governing Equations

Consider an FGP spherically symmetric hollow sphere rested on an elastic foundation on the inner surface and subjected to the multiphysical loading in spherical coordinate system (r,θ,φ). The inner and outer radius of the spheres are assumed to be a and b, respectively. The Winkler-type elastic foundation stiffness is assumed to be k_w and the sphere is subjected to external pressure p_b. The inner and outer surfaces of the sphere are subjected to moisture concentration change m_a and m_b, temperature change ϑ_a and ϑ_b, and electric potential ϕ_a and ϕ_b respectively. The external magnetic field H_φ is also exerted to the sphere.

The constitutive equations are given as:

[1] $$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \beta_{ij}\vartheta - \xi_{ij}m$$
$$D_i = e_{ijk}\varepsilon_{jk} + \in_{ij} E_j + \gamma_i\vartheta + \chi_i m$$

In which σ_{ij}, D_i, ε_{kl}, E_k, ϑ, and m are, respectively, stress, electric displacement, strain, electric field, temperature change, and moisture concentration change; C_{ijkl}, e_{kij}, $\in_{ij}$, β_{ij}, γ_i, ξ_{ij}, and χ_i are elastic, piezoelectric, dielectric, thermal stress, pyroelectric, hygroscopic stress, and hygroelectric coefficients, respectively.

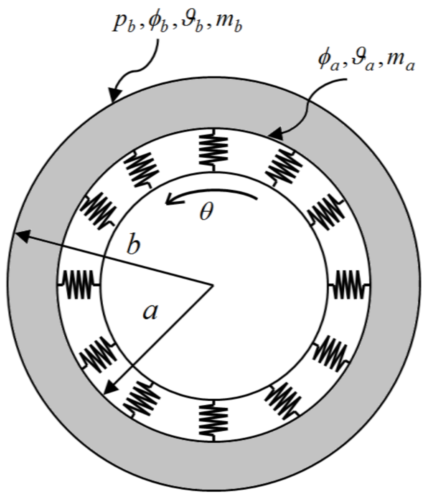


Figure1: An FGP hollow sphere resting on elastic foundation

In addition, strain-displacement relation for small deformation and the quasi-stationary electric field equation are specified as:

[2] $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ E_i = -\phi_{,i}$

where u_i and ϕ represent, respectively, displacement component and electric potential. Using Eqs. 1 and 2 for transversely isotropic and radially polarized materials, the following constitutive equations in the spherical coordinate could be written:

[3] $\sigma_{rr} = c_{33}u_{,r} + 2c_{13}\frac{u}{r} + e_{33}\phi_{,r} - \beta_1\vartheta - \zeta_1 m$

$$\sigma_{\theta\theta} = c_{13}u_{,r} + c_{11}\frac{u}{r} + e_{31}\phi_{,r} - \beta_3\vartheta - \zeta_3 m$$

$$D_r = e_{33}u_{,r} + 2e_{31}\frac{u}{r} - \in_{33}\phi_{,r} + \gamma_1\vartheta + \chi_1 m$$

in which $c_{pq} = C_{ijkl}$, $e_{pl} = e_{kij}$, $\beta_p = \beta_{ij}$, and $\xi_p = \xi_{ij}$ $(i, j, k, l = 1,2,3;\ \ p, q = 1,2,\ldots,6)$ and $u = u_r$ is the displacement in the direction and $\sigma_{\varphi\varphi} = \sigma_{\theta\theta}$.

The quasi-static equation of motion and Maxwell equation for an electrically conducting hygrothermopiezoelectric sphere are written as:

[4] $\sigma_{rr,r} + \frac{2}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \mu H^2(u_{,rr} + 2\frac{u_{,r}}{r}) = 0$

$$D_{r,r} + \frac{2}{r}D_r = 0$$

where μ is magnetic permeability and H stands for H_φ. To investigate the effect of temperature and moisture on the multiphysical behavior, the steady-state temperature and moisture concentration should be obtained by solving the following Fourier heat conduction and moisture diffusion equations:

[5] $\frac{1}{r^2}\left(r^2 k^T \theta_{,r}\right)_{,r} = 0,\ \frac{1}{r^2}\left(r^2 k^C m_{,r}\right)_{,r} = 0$

In which, k^T and k^C are thermal conductivity and moisture diffusivity coefficients, respectively.

The material properties of the FGP sphere vary along the radial direction according to the power-law formulation as follows:

[6] $c_{pq}(r)=c_{pqo}\left(\frac{r}{b}\right)^{2n_1}$, $e_{pq}(r)=e_{pqo}\left(\frac{r}{b}\right)^{2n_1}$, $\in_{pq}(r)=\in_{pqo}\left(\frac{r}{b}\right)^{2n_1}$, $\mu(r)=\mu_o\left(\frac{r}{b}\right)^{2n_1}$, $\beta_p(r)=\beta_{po}\left(\frac{r}{b}\right)^{2(n_1+n_2)}$

$$\zeta_p(r)=\zeta_{po}\left(\frac{r}{b}\right)^{2(n_1+n_2)},\; k^T(r)=k_o^T\left(\frac{r}{b}\right)^{2n_3},\; k^C(r)=k_o^C\left(\frac{r}{b}\right)^{2n_3}\; (p,q=1,2,\ldots,6)$$

where n_1, n_2, and n_3 are non-homogeneity indices and subscript 'o' indicates the material properties at the outer surfaces of the sphere.

3 Solution Procedure

The governing differential equations are obtained as follows using Eqs. (3), (4), and (6):

[7]

$$(c_{33o}+\mu_o H^2)r^2u_{,rr}+((2n_1+2)c_{33o}+2\mu_o H^2)ru_{,r}+(2(2n_1+1)c_{13o}-2c_{11o})u+e_{33o}r^2\phi_{,rr}+((2n_1+2)e_{33o}-2e_{31o})r\phi_{,r}$$
$$-\beta_{1o}\left(\frac{r}{b}\right)^{2n_2}r^2\vartheta_{,r}-2((n_1+n_2+1)\beta_{1o}-\beta_{3o})\left(\frac{r}{b}\right)^{2n_2}r\vartheta-\zeta_{1o}\left(\frac{r}{b}\right)^{2n_2}r^2m_{,r}-2((n_1+n_2+1)\zeta_{1o}-\zeta_{3o})\left(\frac{r}{b}\right)^{2n_2}rm=0$$

$$e_{33o}r^2u_{,rr}+2\left((n_1+1)e_{33o}+e_{31o}\right)ru_{,r}+2(2n_1+1)e_{31o}u-\in_{33o}r^2\phi_{,rr}-2(n_1+1)\in_{33o}r\phi_{,r}+\gamma_{1o}\left(\frac{r}{b}\right)^{2n_2}r^2\vartheta_{,r}$$
$$+2(n_1+n_2+1)\gamma_{1o}\left(\frac{r}{b}\right)^{2n_2}r\vartheta+\chi_{1o}\left(\frac{r}{b}\right)^{2n_2}r^2m_{,r}+2(n_1+n_2+1)\chi_{1o}\left(\frac{r}{b}\right)^{2n_2}rm=0$$

To simplify the solution procedure, the following parameters are used:

[8] $\alpha=\frac{c_{11o}}{c_{33o}}$, $\delta=\frac{c_{13o}}{c_{33o}}$, $\delta^*=\frac{c_{12o}}{c_{33o}}$, $\beta=\frac{e_{31o}}{e_{33o}}$, $\nu=\frac{\chi_{1o}c_{33o}}{e_{33o}\zeta_{1o}}$, $\gamma=\frac{\in_{33o}c_{33o}}{e_{33o}^2}$, $\lambda=\frac{\gamma_{1o}c_{33o}}{e_{33o}\beta_{1o}}$, $\eta=\frac{\beta_{3o}}{\beta_{1o}}$, $\varsigma=\frac{\xi_{3o}}{\xi_{1o}}$

$$\Omega=\frac{\mu_o H^2}{c_{33o}},\; \rho=\frac{r}{a},\; M=\frac{\xi_{1o}}{c_{33o}}m,\; \Theta=\frac{\beta_{1o}}{c_{33o}}\vartheta,\; \Phi=\frac{e_{33o}}{c_{33o}}\phi$$

Using the parameters defined in Eq. 8 and changing the non-dimensional variable ρ with s by $\rho=e^s$, Eq. 7 could be written as:

[9]

$$(1+\Omega)\ddot{u}+(2n_1+2(1+\Omega)-\Omega-1)\dot{u}+2((2n_1+1)\delta-\alpha)u+\ddot{\Phi}+(2n_1-2\beta+1)\dot{\Phi}$$
$$-a\iota^{-2n_2}e^{(2n_2+1)t}\dot{\Theta}-2a\iota^{-2n_2}(n_1+n_2+1-\eta)e^{(2n_2+1)t}\vartheta-a\iota^{-2n_2}e^{(2n_2+1)t}\dot{M}-2a\iota^{-2n_2}(n_1+n_2+1-\varsigma)e^{(2n_2+1)t}M=0$$
$$\ddot{u}+\left(2n_1+2\beta\right)+1\big)\dot{u}+2(2n_1+1)\beta u-\gamma\ddot{\Phi}-(2n_1+1)\gamma\dot{\Phi}+a\iota^{-2n_2}\lambda e^{(2n_2+1)t}\dot{\Theta}+2a\iota^{-2n_2}(n_1+n_2+1)\lambda e^{(2n_2+1)t}\Theta$$
$$+a\iota^{-2n_2}\nu e^{(2n_2+1)t}\dot{M}+2a\iota^{-2n_2}(n_1+n_2+1)\nu e^{(2n_2+1)t}M=0$$

in which the overdot stands for the differentiation with respect to *s* and $\iota=\frac{b}{a}$. Having the temperature and moisture concentration distributions, the governing differential equations 9 can be solved analytically [Akbarzadeh and Chen, 2013]. Accordingly the radial displacement and electric potential are obtained as follows:

[10] $u=A\rho^{r_1}+B\rho^{r_2}+C\rho^{r_3}+K_1\rho^{2n_2-2n_3}+K_2\rho^{2n_2+1}$

$$\Phi=b_1A\rho^{r_1}+b_2B\rho^{r_2}+b_3C\rho^{r_3}+K_3\rho^{n_2-n_3}+K_4\rho^{2n_2+1}+D$$

In which, *A*, *B*, *C,* and *D* are integration constants and the other coefficients are defined by Akbarzadeh and Chen [2013]. Also, the following non-dimensional stress, displacement, and electric potential are defined for convenience:

[11] $\Sigma_{rr}=\frac{\sigma_{rr}}{c_{33o}}$, $U=\frac{u}{a}$, $\Phi_1=\frac{\Phi}{a}$

The integration constants are obtained by satisfying the following electromechanical boundary conditions:

[12] $\Sigma_{rr}(1)=K_W U_a$, $\Sigma_{rr}(\iota)=\Sigma_{rrb}$, $\Phi_1(1)=\Phi_{1a}$, $\Phi_1(\iota)=\Phi_{1b}$

in which $K_W = \dfrac{ak_w}{c_{33o}}$ and subscript '*a*' and '*b*' correspond to the inner and outer surfaces.

4 Results

In this section, some of the results are illustrated for uncoupled magneto-hygrothermopiezoelectric analysis of an FGP hollow sphere rested on elastic foundation on the inner surface. The effects of multiphysical loadings of pressure, electric potential, temperature change, and moisture change on the structural behavior of the FGP sphere are investigated. The material properties of the sphere are assumed the same as those specified by Akbarzadeh and Chen [2013].

The effect of hygrothermal loading on the multiphysical responses is particularly investigated. Figure 2 shows the temperature distribution through the thickness of an FGP hollow sphere. The inner surface of the sphere is kept at reference temperature and moisture concentration, while the temperature and moisture concentration are increased on the outer surface. Since the temperature and moisture concentration change similarly according to steady state Fourier heat conduction and Fickian moisture diffusion, the temperature change is only depicted in Figure 2. As depicted in this figure, increasing the temperature on the outer surface raises the temperature within the sphere.

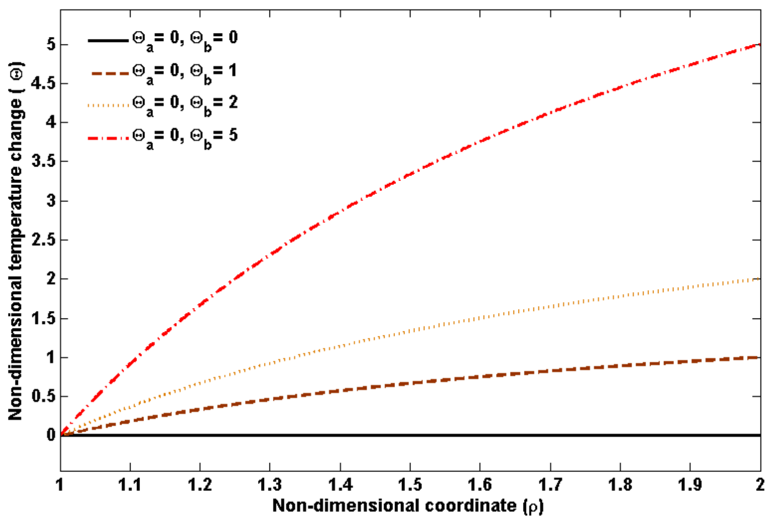
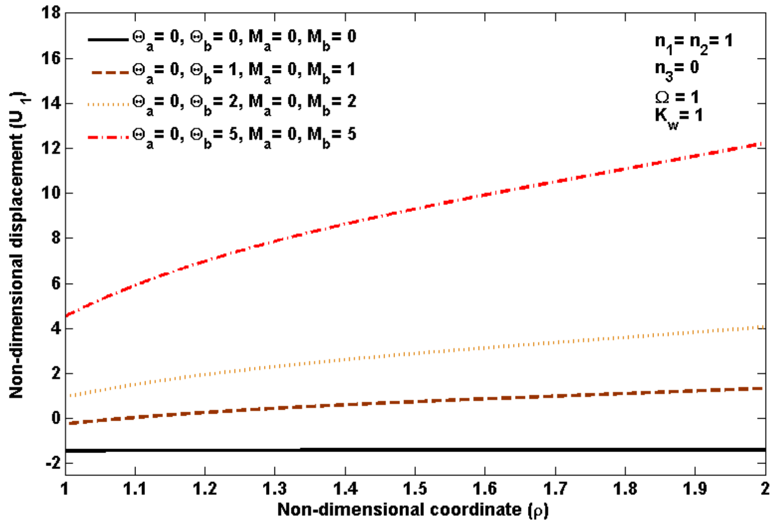


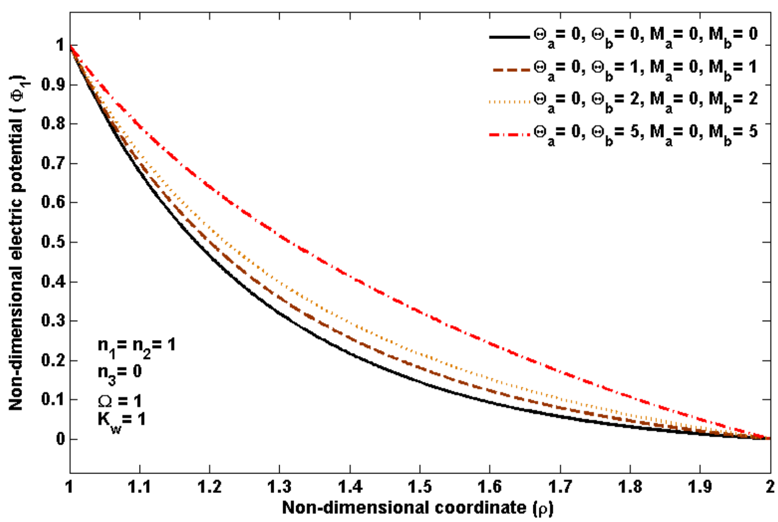
Figure 2: Temperature change within the FGP sphere

Regarding the hygrothermoelectroelastic interaction, the variation in temperature and moisture concentration changes the radial displacement and electric potential as well as radial and hoop stresses throughout the sphere. Figure 3a through 3d reveal the influence of hygrothermal loading on the structural behavior of an FGP hollow sphere with $n_1 = n_2 = 1$ and $n_3 = 0$ rested on an elastic foundation with $K_W = 1$ and placed in a constant magnetic field $\Omega = 1$. The FGP sphere is subjected to the following electromechanical boundary conditions: $\Sigma_{rr}(\iota) = -1$, $\Phi_1(1) = 1$, $\Phi_1(\iota) = 0$. As seen in Figure 3a and 3b, increasing the hygrothermal loading on the outer surface raises the radial displacement and electric potential within the sphere. Furthermore, higher hygrothermal loadings results in greater radial stresses and make the compressive radial stresses on the inner surface tensile, as depicted in Fig 3c. As shown in Fig 3d, enhancing the hygrothermal loading vanishes the compressive hoop stresses throughout the sphere and make them tensile.



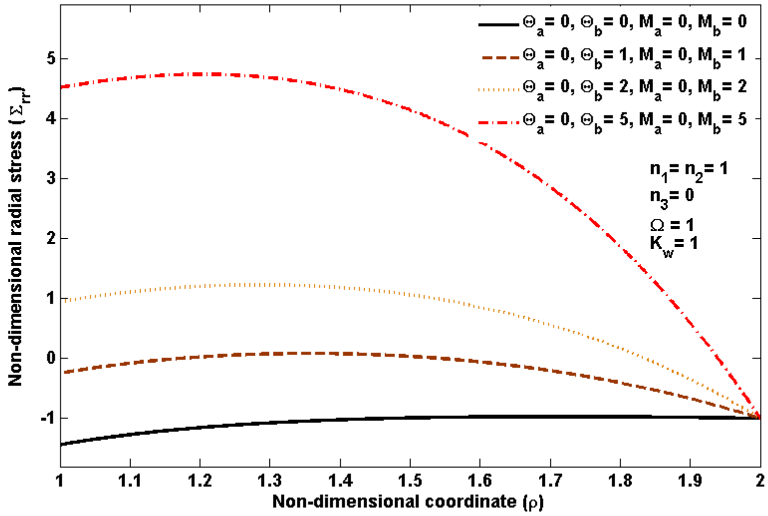
Θa= 0, Θb= 0, Ma= 0, Mb= 0
Θa= 0, Θb= 1, Ma= 0, Mb= 1
Θa= 0, Θb= 2, Ma= 0, Mb= 2
Θa= 0, Θb= 5, Ma= 0, Mb= 5
n1= n2= 1
n3= 0
Ω = 1
Kw= 1
Non-dimensional displacement (U 1)
Non-dimensional coordinate (ρ)

(a)

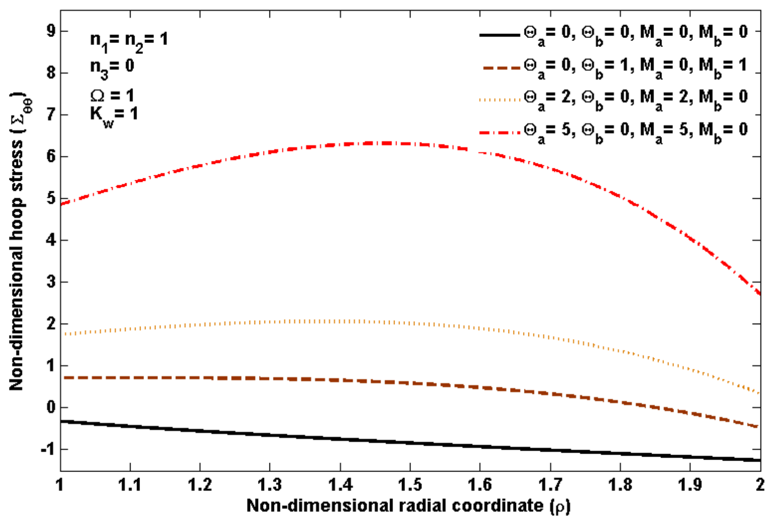


Θa= 0, Θb= 0, Ma= 0, Mb= 0
Θa= 0, Θb= 1, Ma= 0, Mb= 1
Θa= 0, Θb= 2, Ma= 0, Mb= 2
Θa= 0, Θb= 5, Ma= 0, Mb= 5
n1= n2= 1
n3= 0
Ω = 1
Kw= 1
Non-dimensional electric potential (Φ1)
Non-dimensional coordinate (ρ)

(b)



(c)



(d)

Figure 3: Effect of hygrothermal loading on: (a) radial displacement, (b) electric potential, (c) radial stress, and (d) hoop stress distributions

5 Conclusions

The uncoupled magneto-hygrothermoelectroelastic responses of an FGP sphere was investigated. The constitutive, conservation, and potential field equations for the uncoupled multiphysical analysis were introduced. The governing differential equations in terms of displacement and electric potential with temperature, moisture, and Lorentz force effects were obtained in the form of second-order differential equations. The governing equations were found analytically and closed-form solutions for displacement, electric potential, and stress components were found. The numerical results revealed that the hygrothermal loading along with external magnetic field and elastic foundation could change the structural behavior of the FGP significantly.

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