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Bayesian Updating and Structural Model Validation of Bridges based on Ambient Vibration Tests

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Abstract: Analytical seismic fragility analysis of bridges provides a powerful tool to identify weak structural components and to assess the reliability of existing structures. To decrease uncertainties relative to structural models, analysts often acquire field data to validate their models. Ambient vibration test provides a nonintrusive and inexpensive procedure to assess the dynamic behaviour of full-scale structures. The objective of this paper is to utilize Bayesian methods as a robust approach to update input variables of the model and to assess the confidence and the uncertainties involved in a structural model based on ambient vibration test results. In this study, the total mass of the structure, concrete strength and rotational stiffness of the bearings are considered as variables which are updated based on the first natural frequency of the structure. This approach is applied to a typical 5-span concrete bridge as a case study.

1 Introduction

Many developed countries are struggling with the issue of deteriorated bridges in their transportation system due to aging and damage caused by increased magnitude and volume of vehicular loads. Moreover, seismic deficiencies of existing bridges must be assessed and addressed. In both cases, these require some assessment that often requires a model that offers an accurate representation of current condition and response of the bridge.

Ambient vibration testing has recently become a popular method for assessing the dynamic behaviour of full-scale structures. This test is used to estimate the natural frequencies and mode shapes of the structure. Ambient vibration surveys are non intrusive because no excitation equipment is needed since the natural or environmental excitations are used which translates into minimal interference with the normal function of the structure. The motivation for performing this test is to update input structural variables, model validation and locate probable major deficiencies in the structure.

In this paper, first the Finite Element model of the bridge and the ambient vibration survey results are presented. Then, validation of the model is performed using two stochastic approaches: Classical hypothesis and Bayesian hypothesis testing. Updating of parameters for input variables and major deficiency detection are also discussed in the Bayesian hypothesis test section.

2 Finite Element Modeling and ambient vibration test

The bridge under study has three lanes as well as a bicycle path and a pedestrian walkway. The heavy pedestrian walkway and the light bicycle path result in a non-uniform weight distribution of the bridge section and are expected to affect the natural frequencies, dominant modes and generally the behaviour of the structure.

The bridge is designated as a lifetime structure and needs to meet the highest standards in terms of reliability. This Bridge consists of 5 spans with a total length of 232 meters. The superstructure consists of a concrete deck with 0.203 m thickness which is supported by 5 steel girders with varying depth. Except for the bearing at pier 2, which is a low type fixed bearing, all other bearings at the piers and abutments are high steel bearings. The piers located in the river bed are supported by regular footings on hard rock.

The bridge is modeled with finite elements with the computer program SAP2000. A three dimensional view of the model is shown in Figure 1. The bridge deck and the girders are modeled with 4-node shell elements. The piers are modeled with nonlinear multi layered shell elements. Cap beams are modeled with beam elements and the bearings and the abutments are modeled using Nllink elements which have six independent springs, one for each of six deformational degrees of freedom (SAP2000, 1996). Non-confined concrete material behaviour is assumed to model the behaviour of the piers (Mander, 1988). The behaviour of bearings is determined by finite element simulation in ABAQUS program. And the behaviour of the abutments is determined based on the proposed model by Shamsabadi et al. (2007) for granular soils. The behaviour of abutments and bearing are shown in Figure 2.

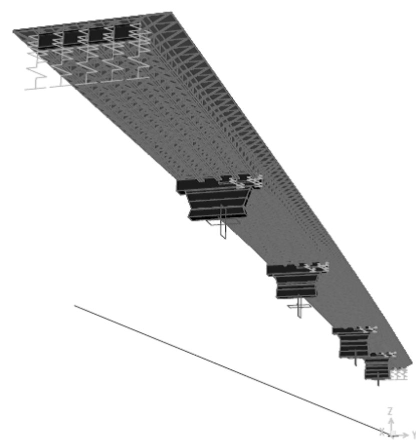
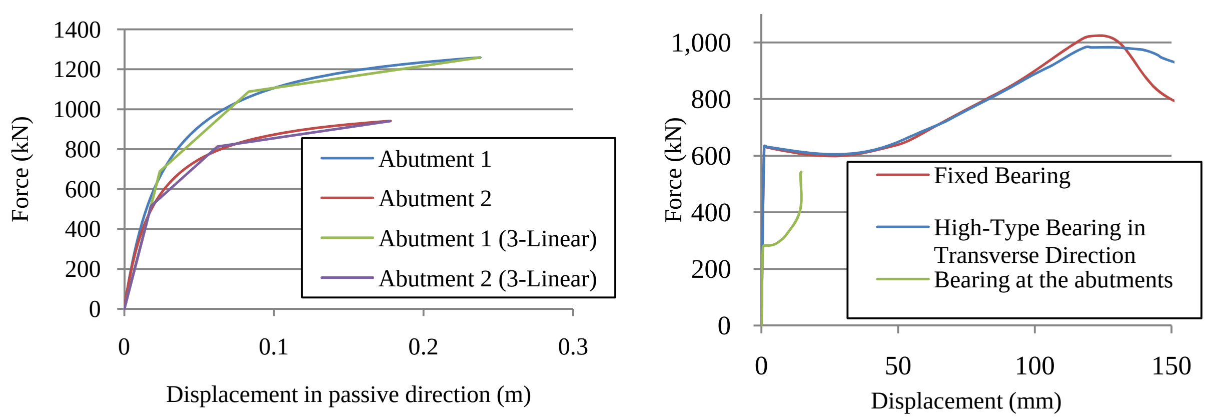


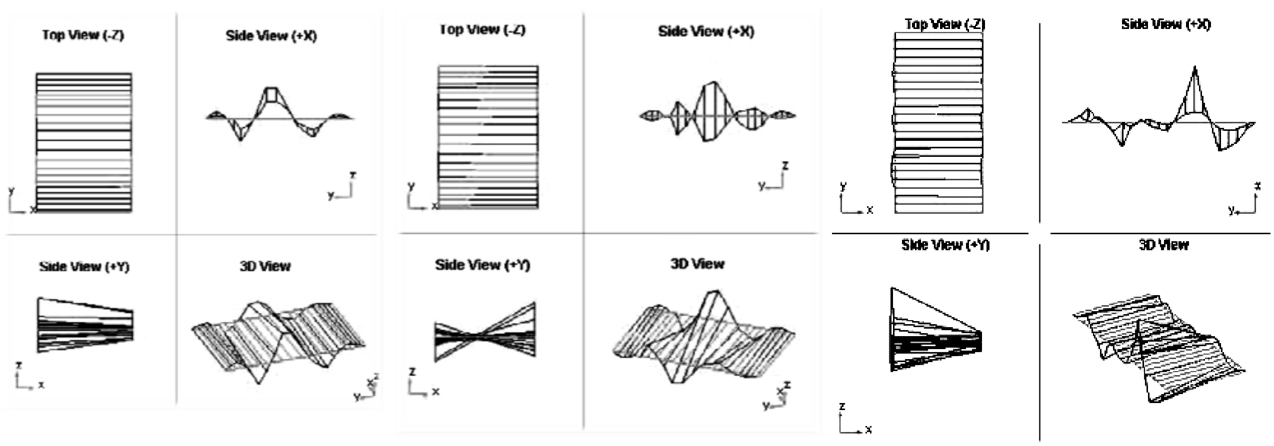
Figure 1: Three Dimensional view of Finite Element model



(a) (b)

Figure 2: Force-Displacement relationship in a) Abutments b) Bearings

Ambient vibration test was performed using 6 sensors, 4 of which were roving sensors and 2 reference sensors. A total of 11 setups were used to cover 21 points on each side of the bridge and the movements in 3 directions were recorded. The natural frequencies and the mode shapes were obtained using the EFDD (Enhanced Frequency Domain Decomposition) analysis. Figure 3 presents the first three mode shapes of the structure and Table 1 compares natural periods of the bridge obtained with the ambient vibration test and with the finite element model for the first twenty natural modes of the bridge which correspond to 76%, 73% and 50% mass participation factors in x, y and z directions, respectively. It is noted that the natural modes of the bridge are combined with the torsion resulted from non-uniform weight distribution of the bridge section.



(a) (b) (c)

Figure 3: Results of EFDD analysis for the (a) first, (b) second and (c) third mode shapes of the bridge

Table 1: Comparison of natural periods obtained by ambient vibration test and the Finite Element model

Mode number		1	2	3	4	5	6	7	8	9	10
Measured Period	Sec	0.69	0.63	0.46	-	0.40	0.36	0.30	0.28	0.26	0.26
Model Period	Sec	0.70	0.62	0.51	0.42	0.39	0.37	0.31	0.28	0.26	0.26
Error	(%)	0.76	1.13	10.34	-	2.98	1.62	3.71	1.41	0.86	1.57

Mode number		11	12	13	14	15	16	17	18	19	20
Measured Period	Sec	-	0.21	0.19	0.18	0.16	0.15	0.14	0.13	0.11	0.10
Model Period	Sec	0.23	0.20	0.20	0.18	0.18	0.14	0.13	0.13	0.13	0.12
Error	(%)	-	1.37	1.48	1.66	13.21	3.74	6.04	2.85	10.45	24.16

3 Finite Element Model Validation

Two stochastic approaches are used in this study to validate the finite element model of the bridge structure. These approaches can be utilized for either single or multiple observations. The first approach is based on classical hypothesis tests and demonstrates the confidence level of the model. However, this approach focuses on rejecting incorrect models. Conversely, the second approach focuses on accepting appropriate models using Bayesian hypothesis testing.

In this paper, concrete density, concrete strength and rotational stiffness of the bearings are considered as updating parameters and the first natural period of the structure is considered as the single experimental modal data point. It is noted that the proposed methodology is also applicable on a vector of updating parameters i.e. a set of natural periods or modal shapes.

3-1 Classic Hypothesis Approach

Hills and Trucano (1999) stated that if an experiment falls inside a given confidence bound of the predicted model, the experiment and the model are consistent; otherwise the model will be rejected. This test is the foundation of the methods which reject incorrect models.

An uncertainty propagation technique is used to evaluate confidence bounds of the predicted model. Hence, the probabilistic model of the first natural period of the structure is evaluated through Monte Carlo Simulation (MCS). However, since performing MCS of a finite element model is not computationally feasible, MCS is performed on a metamodel which is developed using Response Surface Method (RSM) as suggested by Chen et. al. (2004).

In this study, concrete density and concrete strength are assumed to have normal distributions. The prior distribution variables are presented in Table 2. For each of the mentioned variables, the range of mean value minus standard deviation to mean value plus standard deviation is uniformly divided into 7 points for the full factorial design of natural period of the structure. The rotational stiffness of the bearings is considered as either free or fixed to detect frozen bearings. Hence, the structural results are provided for two models of M_1 and M_2 corresponding to free rotation and fixed rotation of the bearings. This approach is also applicable to detect other types of major deficiencies in the bridge i.e. shattered concrete or lack of prestress.

The histogram and consequently the distribution of the predicted first natural period of the structure are obtained from a MCS which is performed with 1.000,000 samples for each model of bearing stiffness. The predicted natural period has a normal distribution with the mean value of 0.662 sec and standard deviation of 0.023 sec for the first model and 0.647 sec and 0.023 sec for the second model. The histogram of the response is shown in Figure 4.

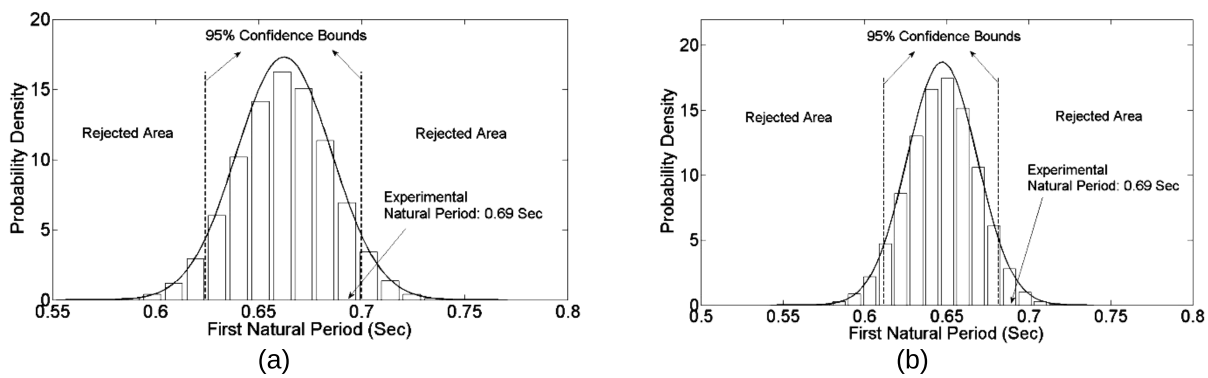


Figure 4: Model Validation using classic hypothesis approach (a) Free rotation at the bearings (b) Fixed rotation at the bearings

According to the results of MCS, the observation of 0.69 Sec corresponds to 92.2% and 98.2% confidence limit in the first and second model respectively. So, if the significant level is set at 95% confidence interval, the experiment fails to reject the first model while the second model is rejected by this test.

3-2 Bayesian Approach

The classic hypothesis approach provides a powerful method to represents the confidence of the model and to reject incorrect models but it does not validate or accept the model. Hence, the Bayesian approach is also used in this study.

According to Mahadevan and Rebba (2004), a model is accepted if the observation favors the model. In other words, if the probability density of the predicted value increases as the condition of the experimental data, the model will be acceptable. Mahadevan and Rebba (2004) and Rebba and Mahadevan (2006) demonstrated that the Bayes factor can be calculated from equation 1:

[1] $$B(x_0) = \frac{f(x|y)}{f(x)}|_{x=x_0}$$

In which x_0 is the predicted value, f(x) represents the prior Probability Density Function (PDF) and f(x|y) represent posterior PDF. It is noted that the $B(x_0)$ higher than unity validates the model. In this study, the posterior PDF is obtained from a MCS similar to the one of previous part considering updated input variables. Input variables- concrete density and strength- are updated as shown in equation 2:

[2] $$f(\theta_i|y) = \frac{L(y|\theta_i).f(\theta_i)}{\int_{\theta_i} L(y|\theta_i).f(\theta_i)\ d\theta_i}$$

In which θ_i represents the input variable, $L(y|\theta_i)$ is the likelihood of the observation in the prior system and $f(\theta_i)$ represents the prior PDF of the input. The PDF of input variables are updated based on equation 2 using MCS method. In addition, in order to demonstrate the probability of having frozen bearings in the bridge, an equal prior probability is assigned to each model of free and fixed bearings and the posterior probability is calculated based on Bayesian equation as presented in equation 3 (Zhang and Mahadevan (2000).

[3] $$P(M_i|y) = \frac{P(M_i).\int_{\theta_i} L(y|\theta_i).f(\theta_i|M_i)\, d\theta_i}{\sum P(M_i).\int_{\theta_i} L(y|\theta_i).f(\theta_i|M_i)\, d\theta_i}$$

By evaluating $P(M_i|y)$ in equation 3 using MCS results, the posterior probability of the models are equal to 0.808 and 0.192 respectively. So, there is a 0.192 probability of having frozen bearings. This result is consistent with the result obtained in the previous method. Hence, the model with free rotation at bearings will be used for the purpose of updating input variables and model validation in this paper. The PDF of the input variables are updated based on equation 2 using MCS method. The results are shown in Table 2.

The resulting posterior PDF of the first natural period has a normal distribution with the mean value of 0.674 sec and standard deviation of 0.018 KN and is shown in figure 5. The PDF of the prior and posterior at x=0.662 sec is equal to 17.345 and 17.7471 respectively. Hence, the Bayes ratio at the predicted point is equal to 1.023 and the model is validated.

Table 2. Parameters of the input variable distributions in prior and posterior states

		Distribution	Prior		Posterior	
			Mean Value	Standard Deviation	Mean Value	Standard Deviation
Inputs	Concrete density	Normal	24 (kN/m^3)	2.4 (kN/m^3)	26.23 (kN/m^3)	1.85 (kN/m^3)
	Concrete strength	Normal	34.5 (MPa)	6.9 (MPa)	32.15 (MPa)	6.58 (Mpa)
Output	First Natural Period	Normal	0.662 (Sec)	0.023 (Sec)	0.674 (Sec)	0.018 (Sec)

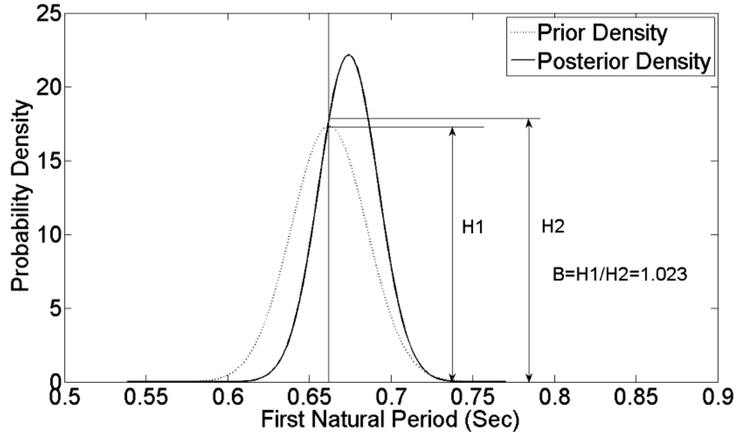


Figure 5: Model Validation using Bayesian hypothesis approach

4 Conclusion

A typical bridge is modeled with Finite Elements as a case study and the model is verified by two stochastic methods. Moreover, based on the first natural period of the structure which is obtained from ambient vibration test, the input variables which are concrete density and concrete strength have been updated using Bayesian method and the probability of a specific major deficiency of having frozen bearings is calculated. It is shown that the Finite Element model is acceptable in the range of elastic behaviour and the probability of having frozen bearings is less than 20%.

The proposed methodology is also applicable for multiple experimental data i.e. a set of natural periods or modal shapes to get more precise results. In addition, the uncertainty of the structural responses in reliability analysis of the structure decrease by using updated input variables. Hence, performing ambient vibration surveys and model validation is suggested to increase the accuracy of structural assessments.

5 Future works

- Performing similar model validation and parameter update using a set of experimental data.
- Evaluating the probability of having various major deficiencies which have significant effect on natural frequencies of the structure.
- Performing the proposed methodology based on load test results to validate the model in a wider range of structural behaviour.

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