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A New Method for Seismic Reliability Analysis of Structures

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Abstract: Seismic reliability methods have to deal with uncertainties associated with earthquake loads. One source of uncertainty comes from ground motion characteristics typically represented by a set of earthquake ground motion records. The other source is the intensity measure. The exceeding probability distribution of seismic demand follows the joint distribution of these two random variables for seismic loads: the ground motion records and the intensity measure. With the distribution of the capacity measure, the probability of structural failure can then be determined. Different methods can be used to evaluate the joint probability distribution. The traditional SAC method determines the probability distribution of seismic demand from conditional distributions given intensity measure levels. The background and mathematical expression of the traditional method is discussed. The new method determines the probability distribution of seismic demand from conditional distributions given ground motion records. The mathematical expression of the new method is presented, which appears to be relatively straightforward. With the new method, the conditional distributions are not affected by adding or removing some ground motion records. Different sampling strategies can be used for different records. The new method determines the exceeding probability of demand from the mean value of selected ground motion records, which is relatively accurate when a few records are used. Numerical procedures of both the traditional and new methods are presented. Both methods were used in analyzing a wood shear wall. The results indicate that both methods can be used in seismic reliability analysis equally.

1 Introduction

Typical buildings and other structures utilize designated yielding elements to dissipate energy imposed by ground acceleration from earthquakes. The energy dissipation in yielding elements is commonly developed through displacement, which exhibits structural lateral movement or drift. The capability of lateral movement for energy dissipation is expressed as ductility. Traditionally, these buildings and structures are designed with the base shear considering empirical-based force modification factors and other prescriptive requirements (NBCC 2010, ASCE 2005). Linear or nonlinear time history analysis may be used for certain structures, such as those with certain irregularities. The code provisions for time history analysis is primarily tied to the base shear method.

The traditional force-based seismic provisions in the building codes are intended to provide life and safety for occupants during large earthquakes. These provisions may not be able to address other performance measures or levels, such as the damage losses that have been observed after the 1994 Northridge earthquake (see for example Bolin and Stanford 1998, Eguchi et al. 1998). A quantitative assessment of the implied reliability level of the designed structures under earthquake loads is needed to address the concerns at targeted performance levels within the life time of the structures. In the past decades, several risk-based procedures were developed for performance based earthquake engineering and design. The traditional fragility analysis determines the exceeding probability of demand conditioned on a specific level of intensity measure (Ellingwood 1994, Rosowsky and Ellingwood 2002, Pang et al. 2009). A fragility analysis does not identify any specific limit state taking into consideration the coupling effect of all random variables. A seismic fragility analysis is commonly used to examine the uncertainty of ground motion records at targeted intensity levels. The fragility analysis is a reasonably accurate method provided that: 1) the source of uncertainties is dominated by earthquake loads; and 2) no uncertainty is associated with targeted intensity levels. The occurrence probability of earthquake intensity measure is determined by seismologists on a regional basis. Determined hazard levels, such as those specified in the building codes (i.e., the design intensity at 2% in 50 years) are commonly used by engineers. With the determined intensity targets, the fragility analysis provides reasonable information about the probabilistic behaviour of structures.

The reliability method in the SAC seismic guideline (Cornell et al. 2002) determines the exceeding probability from the integral of conditional distributions with respect to the intensity measure. If other random variables are considered, multiple integrals can be applied to conditional distributions of these random variables. The advantage of this method is to incorporate all sources of uncertainties into calculation. This method has been widely used in analyzing seismic reliability of structures. It can be used to develop a simplified design format similar to the conventional load and resistance factor design (Cornell et al. 2002). This method was implemented in the response surface method (Zhang and Foschi 2004). It can also be used to determine the probability of failure of components or systems, such as the work by Celika and Ellingwood (2010). This method can also be implemented into design optimization to study the relationship between seismic risk and potential damage/repair cost (Haukaas 2008, Yang et al. 2009). This method will be referred to as the SAC method.

Both the fragility analysis and the SAC method determine demands from nonlinear time history analysis (NTHA) in order to calibrate the implied reliability. Hong et al. (2010) proposed a procedure to calibrate the ductility factor without performing the NTHA. This procedure calibrates seismic design loads to meet the specified target reliability levels for two performance levels.

This study has two main objectives. The first objective is to discuss the relationship between the probability distribution of engineering demand and its two random variables: ground motion records and intensity measure. The second objective is to present a new reliability method similar to, but independent to the SAC method. Numerical procedures of both the SAC method and the new method were discussed. Both methods were used to analyze the seismic reliability of wood shear walls.

2 Demand and Limit State Functions

The probabilistic behavior of structures can be attributed to the uncertainties from capacity and demand. The uncertainty sources include the properties of materials, connections, structural configurations, construction error, earthquake loads and the limitation of human knowledge. The uncertainty due to the inherent randomness is termed aleatoric uncertainty while the uncertainty due to the limitations in human knowledge or in modeling the process is termed epistemic uncertainty. Seismic loads are among the main sources of uncertainty in structural reliability analysis. The coupling effect of the uncertainties from seismic loads is the key interest here.

In seismic probabilistic analysis, earthquake hazard is typically expressed by two variables: a suite of ground motion records and intensity measure. The selected ground motion records represent earthquake characteristics at the observed site. Intensity measure represents the scaled level of ground motion records. Commonly used intensity measures include spectral acceleration, spectral displacement and peak ground acceleration. The selected ground motion records are then scaled to multiple intensity levels. NTHA are performed for all selected ground motion records with multiple scaled intensity levels. This process is called incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002). The peak (interstorey) drift is commonly chosen as the engineering demand measure here.

If r denotes the earthquake ground motion records and im denotes the intensity measure, the drift demand, d, can be expressed as a function, $g(\cdot)$, of the ground motion records and intensity measure, such that :

[1] $$d = g(r, im)$$

Traditionally, the ground motion records and intensity measure are assumed to be randomly independent. The selected ground motion records are discrete and implicitly assumed to be uniformly distributed over the range of earthquake characteristics other than the intensity measure. Let R denote the random variable of the ground motion records, r. It is convenient to express the probability distribution of ground motion records with its density function, $f_R(\cdot)$, shown as:

[2] $$f_R(r) = \begin{cases} f_R(r_j) = 1/N & (j = 1, 2, ..., N) \\ 0 & (j = else) \end{cases}$$

where N = total number of selected ground motion records. It is noted that the probability distribution of ground motion records in Equation 2 is discrete. Therefore, $f_R(r_j)$ may be used instead of $f_R(r)$ in the following discussion.

A number of probability models have been used to model earthquake intensity (see for example Anagnos and Kiremidjian 1988) and the selection of appropriate distributions of the intensity measure is not discussed here. Let IM denote the random variable of the intensity measure, im. The cumulative distribution function (CDF) of the intensity, $F_{IM}(\cdot)$, can be expressed as:

[3] $$F_{IM}(im) = P(IM \le im) = \int_0^{im} f_{IM}(x)dx$$

where $f_{IM}(\cdot)$ = density function of $F_{IM}(\cdot)$.

The ground motion records and intensity are commonly assumed to be uncorrelated. Let D denote the random variable of the drift demand, d. Noting Equation 1, the distribution of drift demand has two random variables: the ground motion records, R, and the intensity measure, IM. Therefore, the drift demand follows a bivariate distribution, shown as:

[4] $$F_D(d) = P(D \le d) = \iint_{g(r,im) \le d} f_R(r) f_{IM}(x) dr dx$$

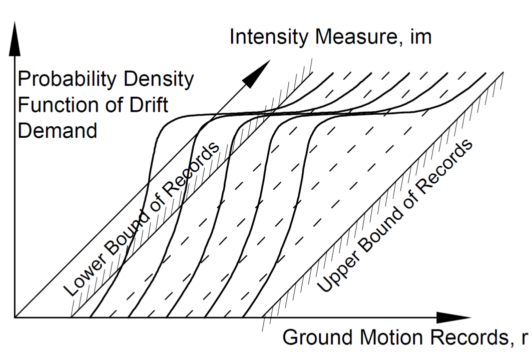

Figure 1: Probability density function of drift demand

The density function of drift demand expressed in Equation 4 is conceptually illustrated in Figure 1. The shape of the surface for the density function depends on the distribution of intensity measure as well as the virtual upper and lower bounds of earthquake ground motion The dashed lines equally spaced between the upper and lower bounds reflect the assumed uniform discrete distribution of ground motion, r. Each curve within every two adjacent dashed lines represents the density distribution of intensity measure for one ground motion record. All curves have the same shape, since all ground motion records are scaled to have the same density function, $f_{IM}(\cdot)$. The density function of drift demand consists of all these curves.

Let C denote the random variable of the drift capacity, c, while $f_C(\cdot)$ denote the density function of drift capacity. The probability of structural failure, P_f, can be expressed as:

[5] $$P_f = P(C < D) = \int_0^{+\infty} f_C(y)[1 - F_D(y)]dy$$

Substituting Equation 4 into Equation 5 yields:

[6] $$P_f = \int_0^{+\infty} f_C(y)[1 - \iint_{g(r,im)\le y} f_R(r) f_{IM}(x) dr dx]dy$$

Theoretically, the double integral of drift demand in Equation 6 can be simplified as two simple integrals, provided that the integration domain, $g(r,im) \le y$, has some specific geometric features. Knowing the nature of numerical analysis for structures exhibiting nonlinear behaviour, it is difficult to predict or identify the geometric features of the integration domain. Therefore, it is difficult, if not impossible, to directly transform Equation 4 to a problem with multiple simple integrals. If other uncertainty sources are considered, the distribution of drift demand in Equation 4 will become a multivariate problem. The integration domain in Equation 6 will be a multi-dimensional "surface" in the Euclidean space, which is almost impossible to find a closed-form solution for nonlinear structures.

This discussion here aims to focus on the coupling effect from earthquake loads for three key random variables: the records, the intensity measure and the drift capacity. All equations can be expanded to consider other sources of uncertainties, such as damage measure and material properties.

3 The SAC Method

For most engineering problems, it is not necessary to obtain a closed-form solution. The SAC method estimates the exceeding probability of drift demand from the conditional distributions given intensity levels (Cornell et al. 2002), shown as:

[7] $$F_D(d) = P(D \le d) \approx \int_0^{+\infty} P(D \le d | IM = x) f_{IM}(x) dx$$

where $P(D \le d | IM = x)$ is the conditional CDF of drift demand, D, not exceeding the value d, given the intensity level of $IM = x$. It should be noted that the transformation from Equation 4 to Equation 7 is based on some engineering judgement. This transformation can only work under certain circumstances as discussed in a number of textbooks such as Ang and Tang (1984). Substituting Equation 7 into Equation 5 gives:

[8] $$P_f \approx \int_0^{+\infty} f_C(y)[1-\int_0^{+\infty} P(D \le y | IM = x) f_{IM}(x)dx]dy$$

If some particular relationship between the ground motion records and intensity measure can be assumed, the probability of failure in Equation 8 can be approximated as an algebra equation. Due to the nature of drift values, a general solution to Equation 8 is typically found using discrete numerical analysis. Numerical solutions can also retain the accuracy of IDA results for the calculation of the probability of failure. A numerical procedure is introduced here. Rewriting Equation 7 in a discrete form gives:

[9] $$F_D(d) \approx \sum_{i=1}^{M} P(D \le d | IM = x_i) f_{IM}(x_i)\Delta x_i$$

where M = number of the intensity levels. The incremental cumulative probability, $f_{IM}(x_i)\Delta x_i$ can be evaluated directly or by any form of Newton Cotes formulae. A simple Newton Cotes formula is used here, shown as:

[10] $$f_{IM}(x_i)\Delta x_i \approx [(F_{IM}(x_{i+1}) - F_{IM}(x_{i-1})]/2 \qquad (i = 1, 2, ..., M)$$

Here $F_{IM}(x_0) = 0$; $F_{IM}(x_{M+1}) = 0$. Substituting Equation 10 into Equation 9 yields:

[11] $$F_D(d) \approx \sum_{i=1}^{M} P(D \le d | IM = x_i)[(F_{IM}(x_{i+1}) - F_{IM}(x_{i-1})]/2$$

Substituting Equation 11 into Equation 5 gives:

[12] $$P_f \approx \int_0^{+\infty} f_C(y)\{1-\sum_{i=1}^{M} P(D \le y | IM = x_i)[(F_{IM}(x_{i+1}) - F_{IM}(x_{i-1})]/2\}dy$$

Equation 12 is a discrete form of Equation 8. It calculates the uncertainty of intensity measure by explicitly evaluating its exceeding probability, $F_{IM}(x_i)$. It does not require any other specific calculation of probability distributions or sampling strategies. The calculation of $P(D \le d | IM = x_i)$ requires that all ground motion records are scaled to the same intensity level, x_i. Equation 12 also permits the intensity measure to be sampled arbitrarily, which is convenient in many applications. If the intensity is sampled such that intensity levels are uniformly distributed in its distribution domain, the incremental cumulative probability, $f_{IM}(x_i)\Delta x_i$ or $[(F_{IM}(x_{i+1}) - F_{IM}(x_{i-1})]/2$ will be close to 1/M. Thus, Equation 12 can be further simplified to avoid the calculation of $F_{IM}(x_i)$.

The SAC method is based on the conditional exceeding probability of drift demand given intensity levels. A general solution to Equation 12 is a numerical analysis with steps shown as:

Step 1) Select a suite of ground motion records. The total number of records is N.

Step 2) Scale the records to multiple intensity levels. The scaled intensity levels do not have to follow its distribution defined by Equation 3. The total number of intensity levels is M.

Step 3) Perform NTHA with the records at scaled intensity levels. Record the peak (interstorey) drift from each NTHA to construct IDA curves.

Step 4) Rank the drift demand values from all records at the same intensity level in order to produce the conditional distribution, $P(D \leq d | IM = x_i)$. Repeat it for all other intensity levels.

Step 5) Determine the probability of failure from Equation 12.

The calculation of the conditional distributions in the SAC method (Equation 8) is based on given intensity, which is consistent with common engineering experience. However, the conditional distributions are established over a set of ground motion records. Depending on the requirement of confidence in the results, a large number of ground motion records may have to be used in these calculations.

4 A New Method Based on Conditional Distributions at Given Ground Motion Records

As mentioned above, the joint probability distribution of drift demand in Equation 4 has two random variables: the ground motion records and the intensity measure. The conditional probability distributions of drift demand in Equation 7 are constructed from all records at certain given intensity levels. Similar to the traditional SAC method, the conditional distributions of drift demand can be constructed from the intensity measure at given ground motion records. Then the exceeding probability of drift demand can be established as:

[13] $$F_D(d) = P(D \leq d) \approx \int_0^{+\infty} P(D \leq d | R = r) f_R(r) dr$$

where $P(D \leq d | R = r)$ is the conditional CDF of drift demand not exceeding the value d, given the ground motion record of $R = r$. The integral sign in Equation 13 implies the variable, r, for ground motion records is expected to be continuous. Since the ground motion records are always discrete samples representing their earthquake characteristics, a practical form of Equation 13 is shown as:

[14] $$F_D(d) = P(D \leq d) \approx \sum_{j=1}^{N} P(D \leq d | R = r_j) f_R(r_j) \Delta r_j$$

Here j denotes the j^{th} ground motion record under consideration. Since the selected ground motion records are uniformly distributed in their representing ground motion characteristics, the values of the probability density function, $f_R(r_j)$, for all records are the same. The intervals between a pair of adjacent ground motion records, Δr_j, are also the same (Figure 1) and:

[15] $$\sum_{j=1}^{N} f_R(r_j) \Delta r_j = 1$$

Substituting Equation 2 into Equation 15 gives:

[16] $f_R(r_j) \Delta r_j = \frac{1}{N} \quad (j = 1, 2, ..., N)$ and substituting Equation 16 into Equation 14 gives:

[17] $$F_D(d) \approx \frac{1}{N}\sum_{j=1}^{N} P(D \le d \big| R = r_j)$$

Finally, substituting Equation 17 into Equation 5 yields:

[18] $$P_f = P(C < D) = \int_0^{+\infty} f_C(y)[1 - \frac{1}{N}\sum_{j=1}^{N} P(D \le y \big| R = r_j)]dy$$

The probability of structural failure expressed in Equation 18 is determined from the mean of the conditional exceeding probabilities of drift demand over selected earthquake records. The conditional probability function, $P(D \le y \big| R = r_j)$, is established from the single IDA curve for the jth ground motion record. The randomness of the conditional exceeding probability comes from the intensity measure. The calculation of conditional exceeding probability does not need to rank the results of drift demand from NTHA. Instead, a mapping procedure can determine the conditional exceeding probability directly. Figure 2 shows the mapping relationship between the drift demand, intensity measure and conditional probability distribution. In this figure, two IDA curves are shown on the right side and the CDF of the intensity is shown on the left side. The determination of the probability not exceeding y given the ground motion record r_j, $P(D \le y \big| R = r_j)$, starts from the drift demand value, y, on the positive side of the abscissa. With the jth IDA curve, the corresponding intensity level can be determined from the ordinate. With the cumulative distribution of the intensity, the cumulative probability at this intensity level is determined from the negative side of the abscissa. This cumulative probability is the conditional exceeding probability, $P(D \le y \big| R = r_j)$, to be determined for Equation 18.

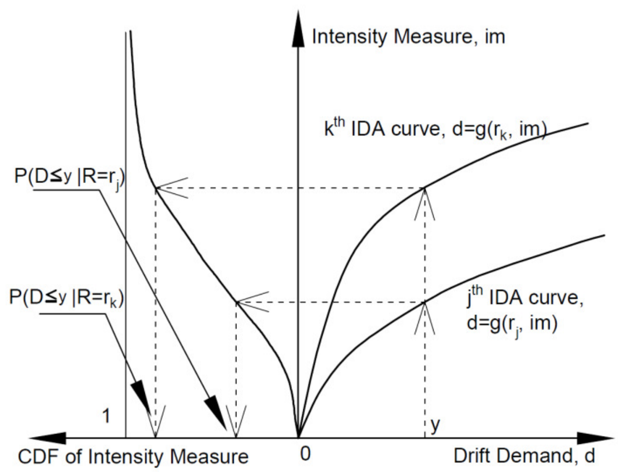

Figure 2: Mapping relationship between drift demand, intensity and conditional distribution

Let $f_{D|r_j}(z)$ denote the density function of $P(D \le z \big| R = r_j)$. Assuming the drift demand in Equation 1 has its inverse function about the intensity measure, $im = d^{-1}(d, r)$, the mapping relationship in Figure 2 can be expressed as:

[19] $$P(D \le y \big| R = r_j) = \int_0^{y} f_{D|r_j}(z)dz = \int_0^{y} f_{IM}[d^{-1}(z, r_j)]\left|\frac{\partial d^{-1}(z, r_j)}{\partial z}\right|dz$$

The function of drift demand in Equation 1 may not have a closed-form solution as the partial derivative of the inverse function of drift demand with respect to intensity measure used by Equation 19 may be unable to be derived explicitly. Thus, it is easier to use the mapping relationship shown in Figure 2 than Equation 19.

The seismic reliability method based on the conditional distributions given ground motion records is analogous to the SAC method. This method has some advantages and disadvantages. First, the computation of the conditional exceeding probability is relatively straightforward, as the calculation of each conditional distribution for one record is independent of other records. One can add or delete some ground motion records without changing the results for other records. Second, using the mean value of conditional exceeding probabilities of drift demand (Equation 19) is the best point estimate over the selected records. The computation is efficient and relatively accurate. Third, the scaled intensity levels for different records do not have to be the same, which allows different sampling strategies to construct different IDA curves. Since structural behaviour for different records may be totally different, flexible sampling strategies for different records is convenient to construct IDA curves. The determination of the conditional exceeding probability shown in Figure 2 starts from a given level of drift demand, which may not coincide with the calculated drift values from NTHA. Iterative NTHA or interpolation may have to be performed to identify the conditional exceeding probability (Gu 2006).

This new method can be used in many different ways. To retain the accuracy of results from NHTA, the numerical procedure to determine the probability of failure is illustrated here:

Step 1) Select a suite of ground motion records. The total number of records is N.

Step 2) Scale the records to multiple intensity levels. The scaled intensity levels do not have to follow the distribution defined by Equation 4. Different records do not have to be scaled to the same intensity levels.

Step 3) Perform NTHA with the records at scaled intensity levels. Record the peak (interstorey) drift from each NTHA to construct IDA curves.

Step 4) Establish the conditional probability distribution, $P(D \leq y | R = r_j)$, for each record with the process illustrated in Figure 2.

Step 5) Determine the probability of failure from Equation 18.

5 Example

A 2.4 m x 2.4 m wood frame shear wall was used in this study (Durham 1998). The shear wall used No. 2 or better 38 x 89 mm Spruce-Pine-Fir dimensional lumber. The end studs and top plates were double members and the bottom plates and interior studs were single members. The sheathing panel was a single 2.4 m x 2.4 m oriented strand board with a thickness of 9.5 mm. The sheathing panel was connected to the framing members with pneumatically driven 50 mm spiral nails. The wall has the interior nail spacing of 300 mm and the edge nail spacing of 75 mm. Hold downs were installed to prevent the overturning of the wall. The dynamic behaviour of the shear wall is simulated with a single-degree-of-freedom system using a nail analogue model (Gu 2006). The visco-damping ratio is 1% of the critical damping about the initial tangential stiffness. The supported seismic mass is 5400 kg.

A suite of 20 ordinary CUREE Woodframe Project records (Krawinkler et al. 2001) were used in the reliability analysis of the shear wall. The intensity measure used is peak ground acceleration (PGA), which is assumed to follow a lognormal distribution with a mean of 0.3g and a coefficient of variation of 0.55. The statistics are consistent with a site design acceleration of 0.86 g (corresponding to a return period of 475 years) and a mean arrival rate of 0.2 (Zhang and Foschi 2004). Choosing this lognormal distribution is to consider the epistemic uncertainty at the targeted intensity level. After performing NTHA for all records at all selected intensity levels, peak drift demand values were recorded and used to construct IDA curves, as shown in Figure 3. Each IDA curve represents seismic response of one selected ground motion record at various PGA levels. There are 20 IDA curves in this figure representing the selected 20 ground motion records. The results of these IDA curves were used to obtain probability distributions of drift demand as discussed in the SAC method and the new method.

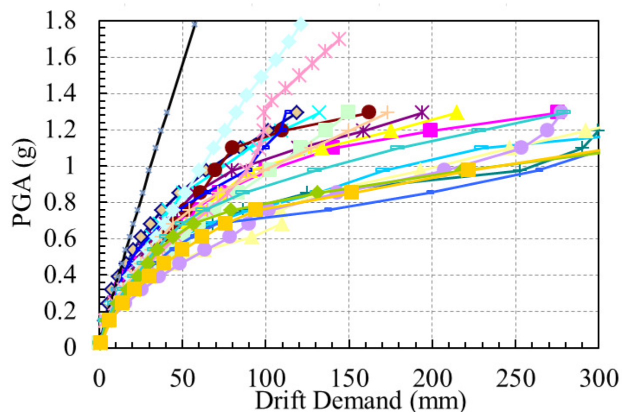

Figure 3: Drift curves from IDA for the selected 20 ground motion records

The drift capacity for the performance level of collapse prevention is 3% of the wall height, i.e., 73.2 mm. No uncertainty is considered for the drift capacity. Reliability analysis was carried out with the SAC method (Equation 12) and the new method mentioned above (Equation 18). The probability of failure was found to be 0.01471 and 0.01962 for the two methods, respectively. These indices are relatively high for seismic loads, probably because of its oversized sheathing panel and relatively low seismic mass. The relatively high indices may be partially attributed to the 3% drift capacity, which is higher than the drift limit of 2.5% specified for life and safety by typical building codes.

The example illustrated here is to demonstrate the application and comparison of both the SAC method and the new method. Other components or systems, intensity measures, targeted hazard levels and uncertainty sources can be implemented in the analysis. The reliability indices from the SAC method and the new method do not show much difference, although the assumptions of the methods are different. It indicates that the new method may be equally useful in seismic reliability analysis.

6 Conclusions

Seismic reliability analysis has to consider the uncertainties from many sources, including earthquake ground motion and intensity. The ground motion records can be treated as uniformly distributed samples of the representing ground characteristics. Different methods can be developed to deal with the coupling effect between the ground motion records and intensity measure. The traditional SAC method calculates the exceeding probability from conditional distributions of drift demand given earthquake intensity levels. The new method presented in this paper is an analogous to the SAC method and determines the exceeding probability of drift demand from conditional distributions given ground motion records. The new method is computationally efficient by allowing different sampling strategies for different records and using mean values of conditional exceeding probability. The results with the new method are relatively reliable, as the method does not require a large number of ground motion records.

Both the SAC method and the new method were used to analyze a 2.4m x 2.4 m shear wall sheathed with an oversized OSB panel. The drift demand of the wall was calculated with a SDOF system subject to ground shaking. A SDOF nonlinear system with parameters calibrated with test results was used in the analysis. The calculated reliability indices from both methods vary from 2.1 to 2.2, which indicate that the new method may be equally useful in seismic reliability analysis.

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