4th Construction Specialty Conference
*4e Conférence spécialisée sur la construction*

Montréal, Québec
May 29 to June 1, 2013 / *29 mai au 1 juin 2013*

# Has the Dust Settled? — A Revisit to the Friedman-Gates Controversy

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**Abstract:** The Friedman-Gates controversy is a famous controversy in the history of scientific research on competitive bidding modeling that started in late 1960s. After many years of frenzied debates, the construction management community did not seem to have reached an agreement. Recent work by the author has completely resolved the controversy. This paper briefly reviews the controversy and reports major findings from the author's recent work, from which a solid conclusion of the Friedman-Gates controversy is drawn.

## 1 Background

The Friedman-Gates controversy (hereafter "the Controversy") refers to the historical controversy in the study of competitive bidding modeling started in late 1960s. The basic decision-theoretic study framework for competitive bidding modeling was established by Friedman (1956) in order to determine an optimal markup size to maximize the company's strategic profit. The probability of the decision-making bidder (hereafter "the Contractor") winning over the competing bidders ("the Competitor(s)") is a key modeling component. The lowest bid is the typical winning criteria used in the modeling. To calculate the probability of winning over multiple Competitors, Friedman suggested a formula based on statistical independence assumption. Gates (1967) adopted the same modeling framework and applied it to construction industry. However, he proposed a different formula to evaluation the probability of winning. The two formulae were so different that many researchers were attracted and tried to figure out which should be actually used. Several of them at various times claimed that they found a final resolution, but those resolutions had never been universally accepted, some later proved to be even wrong. There were a couple of excellent analytical papers in 1970s and 1980s that pointed out the fundamental flaws in Gates' formula. Pitifully, those well-written papers, with their high intellectual sophistication, did not seem to have been widely read, understood and accepted. After entering the new centennial, the construction management community seemed to have lost their interest in the Controversy, although research papers attempting to validate the Gates model still appeared sporadically in diverse journals and conference proceedings.

Meanwhile, some recently published textbooks still present both Friedman and Gates models without telling students and professionals how to address the large discrepancy in the 'optimal' markup or 'best' bid price suggested by the two models. This aggravated practitioners' already distorted belief that quantitative modeling approaches be fruitless.

A couple of years ago, the Controversy attracted the author's attention when he was preparing for an undergraduate course in construction project management. Subsequent research ended up with a beautiful resolution to the Controversy. This paper briefly reviews the controversy and then reports major findings from the author's recent work, from which a solid conclusion of the Friedman-Gates controversy is drawn.

## 2 General Modeling Framework

Before the Controversy is discussed, it is worthwhile to briefly review the general modeling framework that Friedman and Gates took to furnish the best markup to be used in a competitive bidding game. The discussion here is confined to the case where the bidder with lowest bid wins the contract.

The basic idea of competitive bidding model is to determine an optimal markup so as to maximize the expected profit. Note that the *actual* profit cannot be maximized. This is because there are two major uncertainties in determining the actual profit at the time of bidding:

(1) Whether the bid can be won is an uncertain event.
(2) The actual project cost is unknown and the best the bidder can have is the bid estimate.

If the bid is not won, the profit will be zero (given that the estimating cost is negligible or at least considered as a sunk cost). If the bid is awarded, the profit will be the difference between the bid price and the actual cost to be incurred. Yet, the actual cost is unknown till the completion of the project. Therefore, only the *expected* profit can be maximized. To evaluate the expected profit, one can at first assume that actual cost is known and then consider the uncertainty of the actual cost by integrating it out.

So, conditional on a given actual cost $A = a$ (as a notational convention in probability, the upper case letter is used for the random variable and the lower case for its realization), the expected profit is simply the product of the probability of winning, and the difference of the bid price and the actual project cost. In mathematical terms, let $c_0$ denote the cost estimate furnished by the Contractor (the decision maker) for the project under bidding. In this paper, the subscript "0" is used to denote the quantity associated with the Contractor. Let $b_0$ denote the Contractor's bid price, which equals the estimated cost plus a markup, i.e., $b_0 = c_0 + \rho = c_0(1 + m)$ where $\rho$ denotes the markup or profit and $m = \rho/c_0$ is called the markup rate. The probability of the Contractor winning the project at the bid price $b_0$ over the Competitors is denoted by $S$. Therefore, the expected profit with the actual cost $a$, denoted by $\Psi_a$, is expressed as

$$\Psi_a = (b_0 - a) \times S + 0 \times (1 - S) = (b_0 - a)S \qquad (1)$$

This eliminates the first layer of the uncertainty: the chance of winning or losing the game.

Now that the actual cost $A$ is unknown and can be modeled by a continuous random variable with a probability distribution characterized by a cumulative distribution function $F_A(a)$ or alternatively, a probability density function $f_A(a)$, the final expected profit is evaluated by integrating the actual project cost out, i.e.,

$$\Psi = \int \Psi_a \, \mathrm{d}F_A(a) = \int (b_0 - a)S \times f_A(a)\mathrm{d}a \qquad (2)$$

The probability of winning $S$ is a function of the bid $b_0$ and the overall competitiveness. When there is only one competitor, then $S$ is the probability that the Competitor's bid, $B_1$, is greater than the Constractor's bid, $b_0$, i.e., $S = \Pr(B_1 > b_0)$. When there are more than one competitor, it is the probability that $b_0$ is still the lowest, or in other words, the probability that the lowest competing bid, $B_{min} = \min(B_1, \dots, B_n)$, is still greater than $b_0$, i.e., $S = \Pr(B_{min} > b_0)$. Here $n$ is the number of Competitors. Clearly, the probability of winning has nothing to do with the actual project cost and must be independent of $A$. Friedman (1956) treated the independence relationship as an assumption of statistical independence. This is an unnecessary assumption because $S$ and $A$ must be functionally (i.e., logically or mechanistically) independent: bidding occurs way ahead of the competition of a project when the actual project cost will become known. Logical independence governs statistical independence.

With this, the integral in (2) can be simplified as

$$\Psi = \left(b_0 - \int a f_A(a) da\right) S = (b_0 - \mu_A) S \tag{3}$$

where $\mu_A$ denotes the mean value of the random actual project cost $A$. The term "mean value" here is worth a few more words.

Practitioners who are unfamiliar with probability theory may prompt the challenge that the actual project cost is uncertain simply because it is just unknown. They contend that the actual cost should not be treated as a random variable, as the project is not a repetitive event and the random model is lack of statistical basis. To answer this question, we need to first understand that the competitive bidding model is a normative model, not a descriptive model. It is not the Contractor's interest at the time of making the bid decision to predict what will be the exact project cost. It suffices for the decision if the Contractor can quantify the accuracy of the cost estimation. Therefore, although the actual project cost is a unique event which is not random at all, cost estimation is a repetitive process and the relative accuracy of the cost estimate is subject to some statistical law. For a general contractor who wishes to survive in the competitive market for long, it is reasonable to assume that $\mu_A = c_0$. For those who could not satisfy this assumption, they would have disappeared from the market.

The second challenge to the above formulation comes from the modelers. They accept the long-run accuracy argument made above, but they suggest that it must be the unknown actual project cost, rather than the cost estimation, that should be treated as the random variable. Following this line or argument, the expected profit would be expressed as the following:

$$\Psi = \int \left((m+1)c_0 - a\right) S(c_0) f_{C_0}(c_0) \mathrm{d}c_0 \tag{4}$$

where $f_{C_0}(c_0)$ denotes the probability density function of the cost estimate. Eq. (4) cannot be further simplifies because the probability of winning is a function of the integral variable $c_0$. With different value of the cost estimate, the bid price at the same markup rate would vary and hence the probability of winning would also differ. However, the major problem of this formulation is not the mathematical nuisance, but that the formulation confuses a decision variable and a decision parameter. Remember the decision variable of the actual bid decision is the bid price $b_0$. By decision variable it means $b_0$ is a quantity that can be changed freely by the decision maker. In the formulation of (4), $c_0$ is a random variable that is out of the decision maker's control. Moreover, this formulation also artificially mixed up the two uncertainties, which actually are separable and of different nature, as discussed above.

To conclude, the mean value in (3) should be understood as the long-term average accuracy of the estimation of actual project costs. With the assumption that $\mu_A = c_0$, equation (3) can be further simplified as

$$\Psi = ((m+1)c_0 - c_0)S = mc_0 S \tag{5}$$

Since $c_0$ is a decision parameter that is fixed in front of the bid decision, the expected profit can be normalized as

$$\psi = \frac{\Psi}{c_0} = mS(m) \tag{6}$$

$\psi$ can be called the expected profit rate. It is emphasized here that the probability of winning is a function of the markup rate $m$ as it is expressed as

$$S(m) = \Pr(B_{min} > b_0) = \Pr\left(\frac{B_{min}}{c_0} - 1 > m\right) = \Pr(M_{min} > m) \tag{7}$$

where $M_{min} = \min\{M_1', \dots, M_n'\}$ and $M_i' = B_i/c_0 - 1$ is called the apparent markup rate of Competitor $i$. Note that $M_i'$ is not the true markup rate because it is based on $c_0$, the Contractor's cost estimator, not on $c_i$, the Competitor's own cost estimator.

The greatest contribution Friedman made was the proposal to study historical bid data $B_i\ (i = 1, \dots, n)$, relative to the Contractor's cost estimate $c_0$, to evaluate the probability of winning. Specifically, he proposed to study each Competitor's historical bids in terms of the apparent markups (i.e., the ratio of competitor's bid to the contractor's cost estimate for a same project) and fit them with a probability distribution, from which the probability of winning an individual Competitor can be derived. Based on these individual probabilities, one then calculates the overall probability of winning all Competitors. Finally, the optimal markup rate can be easily found by maximize the normalized expected profit $\psi$ in Eq. (6). This is the basic modeling framework Friedman proposed and Gates followed.

## 3 The Controversy

It is the way of aggregating the individual probabilities for the overall probability of winning that caused the Controversy. Denote by $F_i(x)$ the cumulative distribution function of the apparent markup of Competitor $i$. Then the probability of winning over this individual competitor at a given markup rate $m$ can be readily found as $\Pr(B_i > b_0) = \Pr(M_i' > m) = 1 - F_i(m)$. To evaluate the probability of winning over $n$ Competitors, Friedman followed the basic probability principle:

$$S(m) = \Pr(M_{min} > m) = \Pr(\min\{M_1', \dots, M_n'\} > m) = \Pr\{(M_1' > m) \cap \dots \cap (M_n' > m)\} \qquad (8)$$

To proceed, he invoked the assumption of statistical independence among $M_i'$. With this assumption, the probability of winning over multiple Competitors is simplified as the product of the probabilities of winning over each individual Competitors, i.e.,

$$S^F(m) = \prod_{i=1}^{n} \Pr(M_i' > m) = \prod_{i=1}^{n} \big(1 - F_i(m)\big) \qquad (9)$$

The superscript "F" is used to denote the Friedman model.

Gates, driven by the illusion of "share of the work", however, proposed the following equation:

$$S^G(m) = \left[1 + \sum_{i=1}^{n} \frac{F_i(m)}{1 - F_i(m)}\right]^{-1} \qquad (10)$$

The superscript "G" is used to represent for Gates model.

The two formulae are very different not only in model structure, but also in the actual numerical results at a given markup level. In defending his formula, Gates referred to an imaginary bidding scenario in which six bidders are bidding for the same project, each having equal probability of winning over another. According to Gates, the probability of winning of each bidder over the other six ought to be 1/7, as each bidder must have an equal chance of winning the project. However, the Friedman model, under the assumption of independence, predicts a probability of $(1/2)^6$, which is barely 1.6% or 64-to-1 odds against the bidders. Gates' plausible justification for Eq. (10) and the large difference in the formulae invited intense debates that have lasted for several decades.

Several reviews of the Controversy are available, for example, Benjamin and Meador (1979), King and Mercer (1987), Crowley (2000), and more recently, Yuan (2011). In the following, the fundamental flaws of the two models and later developments are summarized.

### *3.1 The Flaw of Friedman's Model*

The flaw of Friedman's model has been very clear at the very beginning. As mentioned above, in order for Eq. (9) to be valid, the apparent markup rates or the apparent markup ratios (defined as the ratio of the Competitor's bid to the Contractor's cost estimate) have to be statistically independent. Whether they are independent or not was not tested until very recently.

### 3.2 Fundamental Flaws of Gates' Model

There are two major fundamental flaws in Gates' model. The first flaw arises from the confusion of perspectives, whereas the second flaw from the misuse of probability theory. These fundamental flaws of Gates' formula were gradually pointed out by Stark (1968), Rosenshine (1972), Fuerst (1976, 1977) and Ioannou (1988).

The symmetry argument Gates used to defend his model was very tricky indeed. Even Friedman was trapped by it. When asked by Gates, Friedman reportedly stated (Gates, 1970): "It is rather obvious that with 7 closely matched competitors the probability of winning will be 1/7." Nevertheless, this argument turned out to be a misuse of perspectives. Ioannou (1988) called it a "fallacy of symmetry". In general, there are two perspectives from which the probability of winning can be calculated. They are the market perspective and the decision-maker's perspective. The market perspective is often used in game theory, or game-theoretic formulation of decision making, to study the market equilibriums at which the Bidders should bid to win the due market share. The market perspective does not help the Contractor to determine the specific size of the markup to win the specific bid, although the Contractor may use the market perspective to review his historical bids. For one specific markup size decision, however, the Contractor has to take the decision-maker's perspective, which is shown in Eq. (6). The major mistake Gates and his followers made in the debates was that they used a market perspective in the calculation of the probability of winning while sticking to the decision-theoretic formulation that Friedman established. To correctly apply the decision models, the problem formulation and the perspective in calculating the probability of winning should be compatible. We cannot use the probability of winning calculated from the market perspective and apply it to a decision-theoretic model. Friedman's model does not suffer from this flaw. In fact, if the market perspective is applied to the imaginary symmetrical bidding, the Friedman model can actually achieve the same answer of $1/n$, as it was pointed out by Fuerst (1976) and reiterated by Ioannou (1988). To answer all challenges based on the symmetry argument, one can simply and firmly state: In deciding the markup size, the Contractor actually does not need to worry about his market share; if he bids as suggested by the model and the other Competitors also bid as suggested by the model, then in the end all bidders will get 'automatically' the due market share.

The second problem of the Gates' model is that even from a market perspective the Gates' model is only conditionally valid. This condition was provided by Fuerst (1976) using a discrete probability example. With the notations in this paper, the condition is expressed as

$$\frac{p_i}{1-p_i} = \frac{S_i}{S_0} \tag{11}$$

where $p_i$ is the probability of the Contractor winning over the individual Competitor $i$, and $S_0$ and $S_i$ are the probabilities of the Contractor and Competitor $i$, respectively, winning the bid. Using the symmetric bidders as an example, because the bidders are equally competitive, the probability of winning the bid is equally distributed; thus $S_i/S_0 = 1$. Meanwhile, the odds of winning over an individual competitor are one by one. Therefore, the condition of (11) is satisfied, and the Gates' model can obtain the desired fraction reflecting the equal market share.

Unfortunately, except for this special case, Gates and the subsequent adherents failed to provide any justification why this condition should be satisfied in general. Gates attempted to justify his development by stating (Baumgarten, Benjamin, & Gates, 1970) that his model "depicts a case of statistically dependent events", which Friedman failed to consider. However, he never provided any mathematical derivation based on rigor probability theory. Benjamin, a Stanford PhD graduate, ever claimed that he had 'derived' Gates' model and concluded that "the [Gates] model is clearly as rational as that used by Friedman although it yields different results" (Baumgarten et al., 1970). But nobody in subsequent debates really trusted his derivation. As R. M. Skitmore, Pettitt, and McVinish (2007) has put, "Benjamin, recognizing that Gates provides no mathematical proof, attempted to rectify the situation but was unable to do so."

The latest attempt to justify Gates model was made by R. M. Skitmore et al. (2007). They maintain that the proportional hazard property of the distribution be a sufficient and necessary condition for Gates' formula. Several comments are worth adding here. First, as many predecessors who tried to 'prove' Gates' model, they misused the market perspective to calculate the probability of winning for a decision-theoretic model. Second, the proportional hazard property of the distribution should be only a sufficient condition because without this condition the Gates' model is also correct for the special case of symmetric bidders as discussed earlier, in which each bidder's bidding pattern can follow any statistical distribution. More importantly, Skitmore et al.'s condition for the Gates' model was derived based on the assumption that the bidding patterns are statistically independent, which is what Gates would definitely avoid.

## 4 Later Development

Motivated by the Controversy, Carr (1982) and Skitmore (1991) developed their own competitive bidding models, both explicitly considering the uncertainty involved in the cost estimate. In order to estimate the accuracy of cost estimate, Carr adopted a set of strong assumptions:

1) All bidders' cost estimates are unbiased;
2) Bidders have the same variance in their cost estimates;
3) The cost estimates and bids are independent and normally distributed; and
4) Variances in cost estimates are substantially greater than variances in markup.

Based on these assumptions, he proposed to evenly split the variance of a bid ratio into two parts: one half for the cost estimate and the other half for the bid. The cost estimate was then taken as a normal distribution with a mean of one and the variance being half of the estimated variance of the bid ratio, and the competitor's bid as another independent normal distribution with the mean being the estimated mean of the bid ratio and the variance equal to the variance of cost estimate. Finally, the probability of winning is expressed as an integral over the cost uncertainty (Carr, 1982).

Skitmore (1991) and M. Skitmore and Pemberton (1994) developed another approach to estimating the statistical parameters for the Contractor's cost estimate and Competitors' bids. Instead of splitting the variances of the bid ratios, Skitmore expressed the mean of the logarithmically transformed data by the sum of bidder's effect and a "contract datum parameter', the latter of which represents the project size and serves as a reference point.

Both the Carr model and the Skitmore-Pemberton model aim at the quantification of the uncertainties in cost estimate. But they also suffer from various defects. The major issue of Carr's model is that those assumptions have been lack of supports from empirical evidence. The even splitting of the variance is very arbitrary, and the independence assumption is made mainly for the sake of mathematical convenience.

The Skitmore-Pemberton model, although making no assumption for the proportion of variances between the cost estimates and bids, requires direct quantification of the uncertainty in the Contractor's cost estimate from historical bid data, which can be a very difficult task. In fact, the statistical method proposed by Skitmore and Pemberton for estimating the variance of the cost estimate cannot obtain the correct value; for more details, refer to Yuan (2011). Moreover, the contract datum parameter, a nuisance parameter, is also hard to estimate. This has made the Skitmore-Pemberton model computationally expensive and data demanding, which might limit its application in real world. On the other hand, although the Skitmore-Pemberton model was called a multivariate model, the multivariate nature was only shown in the location parameters (i.e., the means of the distribution) while the statistical dependence among competitors' bids was not considered. In calculating the probability of winning, the joint distribution function of bids was still expressed as a product of marginal distributions. This means that similar to the Carr model the competitors' bids are also assumed to be independent. More importantly, the Skitmore-Pemberton model uses the formulation frame set in Eq. (4). As it has been discussed in Section 2, this formulation is not proper for the markup size decision.

Two amendments were later proposed to the Skitmore-Pemberton model. Lo (2000) reparameterized the location parameters and further assumed that the variance be an exponential function of the bidder's effect. However, the comparison by Lo (2000) suggested a great difference in the statistics for the bidder's effect between the Skitmore-Pemberton model and Lo's modification. The other amendments to the Skitmore-Pemberton model was proposed by Liu, Wang, and Lai (2005), in which the cost estimate was allowed to be biased. But again, neither of the amendments addressed the preceding formulation issue.

## 5 Recent Development

After tracing back the Controversy and later development, the author recently proposed a genuine multivariate bidding model that explicitly considers the statistical dependence among the bid ratios (Yuan, 2011). Here the bid ratio, also known as the apparent markup factor, is defined as the ratio of the Competitor's bid to the Contractor's cost estimate. Considering that the historical bid data in the real world usually are limited and unbalanced and they includes lots of missing data, he further developed a robust Bayesian statistical procedure based on Markov chain Monte Carlo simulation (Yuan, 2012).

Assumptions associated with the proposed model are as follows:

1) The bid ratios, $X_i$ $(i = 1, \ldots, q)$, follow a multivariate lognormal distribution.
2) The competitors' bidding behavior is stationary; that is, the competitors will bid for the current project in the same manner as they have bid in the past.
3) The Contractor has an unbiased cost estimate.
4) The number of competitors, $n$, and their identity is known for the project under tendering.

A multivariate lognormal distribution has a joint PDF expressed as (Kotz, Balakrishnan, & Johnson, 2000)

$$f(\boldsymbol{x}) = (2\pi)^{-\frac{n}{2}}(\det(\boldsymbol{\Sigma}))^{-\frac{1}{2}}\left(\prod_{i=1}^{n} x_i\right)^{-1} \exp\left\{-\frac{(\log(\boldsymbol{x}) - \boldsymbol{\mu})\boldsymbol{\Sigma}^{-1}(\log(\boldsymbol{x}) - \boldsymbol{\mu})^T}{2}\right\} \quad (12)$$

in which $\boldsymbol{x} = \left(x_1, x_2, \ldots, x_q\right)^T$ is a $n$-dimensional column vector; $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the mean vector and covariance matrix, respectively, of the random vector $\boldsymbol{X}$; and $\det(\cdot)$ denotes the determinant operator of a matrix. The covariance matrix is expressed as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ & & \ddots & \vdots \\ \text{sym.} & & & \sigma_n^2 \end{bmatrix} \quad (13)$$

In shorthand, we denote $\boldsymbol{X} \sim MVLN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ for the multivariate lognormal random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Similar to the univariate counterparts, $\log(\boldsymbol{X})$ follows a multivariate normal distribution, i.e., $\log(\boldsymbol{X}) \sim MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

It is trivial to see that the Friedman model corresponds to a special case of the proposed model when all correlation coefficients are zero. The Carr model may also be considered as a special case of the proposed model, provided that the normal distributions used in the Carr model are replaced by lognormal distributions. With a few mathematical operations it is readily shown that the Carr model can be considered a special case of the proposed model with a strictly patterned correlation matrix. If the variances of Competitors' bids are indeed equal as assumed by Carr, then Carr's model is equivalent to the proposed model with $\sigma_1 = \cdots = \sigma_n$ and $\rho_{ij} = 0.5$ for $i = 1, \ldots, n-1;\ j = 2, \ldots, n$.

The advantages of the proposed model are obvious. First of all, the model provides complete characterization of the uncertainties involved in a bid decision. It characterizes not only the variances, but also the correlations among different bidders. Yuan (2011) explains in detail why the correlation should exists both from a market sharing perspective and from a probabilistic modeling perspective. The case study using real-world data also provides empirical evidence of positive correlations. Moreover, the model structure is adaptabl to the real world. No assumption is made with regards to the relative variations of Competitors' bids and the Contractor's cost estimates. Furthermore, the formulation of the model is self-compatible. It treats the cost estimate $c_0$ as a decision parameter whereas it also considers the effects of the uncertainties in cost estimate on the probability of winning, expected profit and optimal markup. Finally, the proposed model also provides an excellent platform to compare the existing competitive bidding model.

Figure 1 illustrates the effects of correlation. It is shown that the correlation has very significant impact on the probability of winning, the expected profit and the optimal markup. At a given percentage markup, say 4%, the probability of winning can vary from about 5% for $\rho = 0$ to 65% for $\rho = 1$ (Figure 1a). For a fixed number of Competitors, say $n = 5$, the optimal markup can var from about 2% for $\rho = 0$ to 4.5% for $\rho = 1$. Therefore, the actual optimal markup size can be doubled, depending on the value of correlation among the bidders.

Figure 1 also compares the three existing models: Friedman, Gates and Carr models. The Skitmore-Pemberton model is not included in this comparison as it is not a compatible model as discussed above. It is clear that the Gates' and Carr's models are very close, particularly in terms of the optimal markup. This partly explained why there were still people (e.g. Crowley (2000)) advocating the Gates' mode after the Controversy relatively cooled down after 1988.

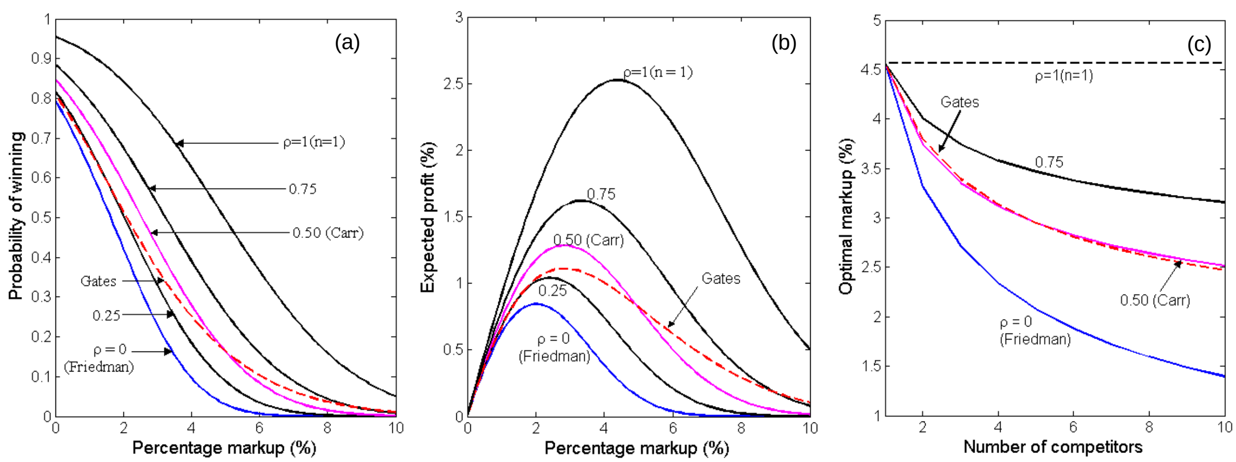


Figure 1: Effects of correlation on (a) the probability of winning with n = 5, (b) the expected profit with n = 5, and (c) the optimal markup percentage.

The only potential issue of the proposed multivariate competitive bidding model is the statistical estimation of the model parameters. The model includes a fairly large number of parameters, including the mean, variances and the correlation coefficients. Reliable estimation of these parameters requires a large number of historical bid data, which might not be available for small to medium-size contractors. To make the situation even worse is that any of two Competitors do not always bid the same projects. This causes a lot of missing data in the database. To clear these practical hurdles, the author developed a Bayesian statistical procedure using modern statistical computation techniques. MATLAB source codes are made available in Yuan (2012).

The case studies by Yuan (2011, 2012) using real-world historical bid data reported empirical evidences of the dependence among the bid ratios. To demonstrate the robustness of the proposed statistical procedure, two different multivariate uniform prior distributions for the correlation coefficients were used in the Bayesian analysis. It was found that the estimated parameters, including the means, standard deviations and correlation coefficients are not insensitive to the selection of the prior distributions. Table 1 shows a sample result from a case study for three Competitors (C1, C2 and C3). The correlation coefficients of the bid ratios among the three Competitors range from 0.3 to 0.75. The other case studies in Yuan (2012) all show significant correlation. Table 1 also reports the very close optimal markups based on the two prior distributions in the Bayesian analysis for the correlated bidding model. The optimal markup of 3% can be used. In contrast, the Friedman, Gates and Carr models would suggest a markup of 2.2%, 3.65%, and 2.82%, respectively. This difference is not small at all for real project.

Table 1: Comparison of parameter estimates for Case 1 with Bidders 1, 55 and 134.

| Prior Distributions | Means | | | Standard deviations | | | Correlation Coefficients | | | Optimal Markup (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 55 | 134 | 1 | 55 | 134 | (1,55) | (1,134) | (55,134) | |
| BU | -0.0517 | 0.0108 | -0.0063 | 0.0187 | 0.0724 | 0.0560 | 0.4582 | 0.7356 | 0.3053 | 3.02 |
| JU | -0.0518 | 0.0116 | -0.0050 | 0.0188 | 0.0718 | 0.0551 | 0.4554 | 0.7308 | 0.2474 | 2.91 |

## 6 Discussions and Conclusions

In competitive bidding, the markup should be high enough to ensure a profit if won, yet not too high to lose the job. Clearly, the markup should be adjusted for the competitiveness of the bidding. The competitiveness depends on the number of Competitors and the lowest possible bid that the Contractor would perceive the Competitors might tender. How to evaluate the competitiveness is a very subtle issue. Statistical method is a good approach to the evaluation of the competitiveness.

The Friedman-Gates Controversy on the probability of winning has been lasting for so many years. Many prominent researchers in construction management and operations research participated in the debates. Now it is the time to close the chapter. It is very clear now that the Gates' model is an engineering approximation at best. The model arrives with a whimsical guess that matches only one special case from a wrong formulation of the problem.

Some construction management professionals tend to believe that quantitative modeling approaches, including critical path methods, are futile. They are short-sighted, of course. However, it is interesting to understand why researchers lost their interest in the study and why professionals lost confidence of the models. Historically, professionals were actually very active in trying to apply the Friedman's model in bidding; this was witnessed by the book by Park and Chapin Jr. (1992), the first edition being published in 1966. However, Park used mainly Friedman's model. As shown in Figure 1, Friedman's model always underestimates the optimal markup. This clearly must have warned any experience and ambitious construction managers not to follow the suggestion based on the Friedman's model. Unfortunately the Gates' model, although providing more realistic markup value, failed to rebuild professionals' confidence. Because of its fundamental theoretical flaws, it effectively sent more smokes in the battlefield of winning professionals' faith in using quantitative approach to improve management efficiency.

From a researcher's perspective, there is an important lesson we should learn from the Controversy. That is, although modeling should start from a certain simplified situation, but any simplification should follow logical rules. Friedman followed the rule very well. He explicitly presented the assumption. Gates also tried to simplify the model, but he did not fully understand the working of probability theory.

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