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A Mathematical Model for Dynamic Site Layout Planning

M. Andayesh[1], F. Sadeghpour[1]
[1] Department of Civil Engineering, Schulich School of Engineering, University of Calgary

Abstract: At any point during a construction project various objects exist on the site (e.g. temporary facilities, batch plants, and tower cranes) that support project activities. Efficient arrangement of these objects on the site, such that it enhances the productivity, safety, and security, is referred to as site layout planning. As the project progresses through its activities, the supporting objects on the site will be subjected to change, and so will the layout of the site. These changes make the optimization of the site layout a challenging task. Considering the actual duration that objects exist on the site will make the layout planning a 4D optimization problem. This paper presents a dynamic layout planning model which uses a mathematical approach to develop optimum layouts. The advantage of mathematical methods lies in their flexibility in defining different constraints and conditions for the project. In the developed model, project boundary conditions such as the actual duration of objects, the workflow between objects, the required and available space, and site-specific constrains are defined in a set of mathematical equations. These equations are then solved using Generic Algebraic Modeling System (GAMS) to generate layouts that are optimized over the duration of the project. A computational example is provided to demonstrate the capabilities of the developed model.

1 Introduction and Background

Construction activities are supported by various objects such as temporary facilities, tower cranes, batch plants, and gravel depot. Site layout planning is the task of determining the optimum location of these objects on the site to ensure safety, productivity, and security of the project (Tommelien et al. 1992, Sadeghpour et al. 2004, Khalafallah and El-Rayes 2011). An efficient layout reduces the cost of material handling and workflow and increases the safety measures on the site (Hegazy and Elbeltagi 1999, Isaac et al. 2012). As the construction activities change, the required objects, and accordingly, their optimum location on the site is subjected to change. Reflecting these changes in the process of searching for optimum layout is the main challenge in dynamic site layout planning (Zouein and Tommelien 1999, Elbeltagi 2001, Andayesh and Sadeghpour 2011).

Different approaches have been taken in site layout planning literature for addressing the changes in object requirement over the duration of the project. *Static* models ignore the time dimension and assume all objects exist on the site for the entire duration of the project (e.g. Tommelein and Zouein 1993, Elbeltagi and Hegazy 2001, Lam et al. 2007, Zhang and Wang 2008, Easa and Hossain 2008). This is not a realistic assumption. Clearly, static models do not provide the most efficient use of space. For instance using the static approach, for the project shown in Figure 1, although the geotechnical lab (object A) leaves the site in the fourth month, batch plant (object C) which enters the site in month 10 can not take its place. As a result, despite its limitations, static approach can offer a simplified method for generating layouts for project with short durations or those with abundant available site space (Andayesh and Sadeghpour 2012).

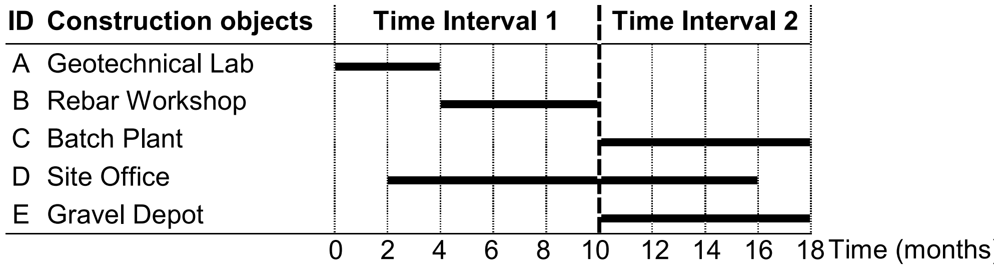

Figure 1: The time lines for construction objects

The next generation of site layout models included the time dimension in the planning process by dividing the project duration into several time intervals and generating an optimum partial layout for each (e.g. Zouein and Tommelein 1999, Elbeltagi et al. 2004, Sadeghpour et al 2006, El-Rayes and Said 2009, Ning et al. 2010, Jiuping and Zongmin 2012). In this *phased* approach, the models can reflect changes in the site from one partial layout to another. However, objects that belong to the same time interval are not allowed to reuse the same space, even if they do not exist on the site at the same time. For instance, if the aforementioned project duration is divided into two time intervals at the end of month 10 as shown in Figure 1, Geotechnical Lab and Rebar Workshop (objects A and B) can not use the same space even though they do not exist on the site at the same time. In addition, partial layouts are often optimized in a chronological sequence. As a result, the locations of objects located in later time intervals are highly influenced by early ones. For instance in Figure 1, the Site Office (object D) has a better chance than the batch plant (object C) to get a desired location since it is located in the first time interval. However, the main limitation of the phased approach is that a collection of separately optimized partial layouts does not necessary result in an overall optimum layout (El-Rayes and Said 2009, Andayesh and Sadeghpour 2013).

To make the most efficient use of site space, a dynamic site layout model is required that considers the actual duration of objects on the site in the optimization process. Such model will allow the reuse of space over time, and at the same time optimizes the location of objects over the duration of the project (4D optimization). The main challenge in dynamic planning is in converging the optimization as the number of variables and constraints gets too large. The authors have previously developed a dynamic model for site layout planning that used minimum total potential energy concept from physics to reach the optimum layout (Andayesh and Sadeghpour 2012 and 2013). While this model was the first to generate layouts that are optimized over the duration of project, it was limited in terms of the type of relationships that could be defined between objects. This paper presents a new dynamic site layout planning model that uses a mathematical approach to define and solve the site layout problem. The advantage of using a mathematical approach is that it provides more flexibility in defining different types of relationships and constraints between objects. As a result, the developed model has better capabilities in reflecting the actual conditions that occur on a construction site. A computational example is provided to demonstrate the capabilities and evaluate the results of the developed model.

2 Model Development

The key point in the developed model is that all the elements and constraints of the problem such as definition of the site boundary, spatial relationships between objects, and the objective function, are represented through a set of equations. These equations, collectively, define the site layout problem. They are then solved as a mathematical optimization problem to determine the optimum arrangement of objects on the site. This section presents how the key components of the developed model such as the objective, constraints, and objects are represented with mathematical equations.

2.1 Site layout objects

Site layout objects are classified into two groups based on their role in the optimization: Site objects (S) and Construction objects (C) (Andayesh and Sadeghpour 2013). Site objects are those that have a known location on the site prior to the start of construction, such as the structure under construction whose location can be determined from the project drawings. Site objects are often permanent and remain on the site after the construction ends. On the other hand, the construction objects are located on the site temporarily to support construction activities. In fact, the aim in site layout planning is to determine the optimum location of these objects on the site. Offices, batch plants, and storage areas are examples for construction objects. In this model objects are represented by their minimum bounding circle to simplify the generated equations. The location of objects and the space they occupy can be determined by the coordinates of the center of their minimum bounding circles and their radius. The following notation is used through this paper to refer to these values:

(X_{Si} , Y_{Si}): coordinates of the center of the Site object i

(X_{Ci} , Y_{Ci}): coordinates of the Construction object i

R_i : Radius of object i

The radius of minimum bounding circles for both type of objects and the center of site objects are known variables, while the coordinates of the center of construction objects are the unknown variables in the layout optimization problem. The efficiency of a layout depends on how well the construction objects are located in the available space on the site.

2.2 Objective

Many of the productivity, safety, and security goals can be expressed using the closeness relationships. In term of productivity, objects that have a large workflow with one another need to be located close to each other to reduce the associated costs. For instance a gravel depot should be located close to the batch plant to reduce the cost of material handling. On the other hand, some objects need to be located far from one another to ensure safety in the site. For example the storage area should be located far from welding workshop to prevent fire hazards. The fitness of a layout in terms of closeness relationships can be measured using the following utility function:

[1] $$UF = \Sigma\, W_{ij}.D_{ij}$$

where W_{ij} represents the workflow between object i and j and D_{ij} reflects the distance between them, which can be determined as:

[2] $$D_{ij} = \sqrt{\left(X_i - X_j\right)^2 + \left(Y_i - Y_j\right)^2}$$

where (X_i , Y_i) and (X_j , Y_j) represent the position of objects i and j, respectively. Closeness weights, which can take positive or negative values, are assigned between pairs of objects. These weights are determined based on the workflow and safety related issues and show how much two objects are desired to be located close to or far from one another (Osman et al. 2003). A larger positive closeness weight shows a significant workflow between two objects. Minimizing the utility function from Eq. 1 results in locating objects with large workflow close to each other. On the other hand, a negative weight indicates that the two objects need to be located far from one another. Minimizing the utility function will provide the optimum location for construction objects on the site based on the defined relationships between pairs of objects.

2.3 Site Boundary Constraint

Clearly, construction objects should be located inside the site boundaries. Therefore, a constraint is required to ensure that this condition is fulfilled. In this model the site boundary is defined by connecting the boundary vertices (B_1 to B_L) to form the border lines of the site. The site boundary constraint is defined using vector calculus in mathematics. Each border line is represented by a vector (Figure 2):

[3] $$\overrightarrow{V_l} = \begin{bmatrix} X_{B(l+1)} - X_{Bl} \\ Y_{B(l+1)} - Y_{Bl} \end{bmatrix} \; for\; l = 1, \dots, L-1$$
$$\overrightarrow{V_L} = \begin{bmatrix} X_{B1} - X_{BL} \\ Y_{B1} - Y_{BL} \end{bmatrix} \; for\; l = L$$

in which V_l is the vector corresponding to border line *l*, and (X_{Bl} , Y_{Bl}) is the coordinates of vertex *l* on the site boundary.

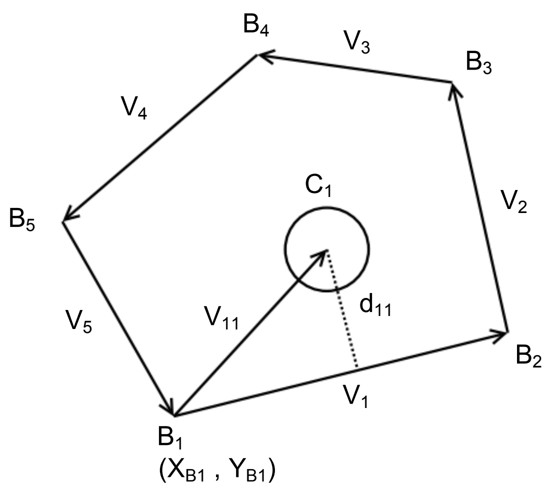

Figure 2: Representing the site boundary constraint

As can be inferred from Figure 2, in order to fulfill the site boundary constraint, the objects should always fall on the left side of the border vectors. In addition, objects need to have a minimum distance equal to their radius from the site boundary which can be formulated as:

[4] $$d_{lj} \geq R_j$$

where d_{lj} is the shortest distance between construction object *j* from border vector *l* (Figure 2), and R_j is the radius of construction object j. This distance (d_{lj}) can be determined from the cross product of border vector V_l and vector V_{lj}. V_{lj} is the vector that connects vertex l of the site boundary to the construction object j. This vector can be determined using the following equation:

[5] $$\overrightarrow{V_{lj}} = \begin{bmatrix} X_{Cj} - X_{Bl} \\ Y_{Cj} - Y_{Bl} \end{bmatrix}$$

where *(X_{Cj} , Y_{Cj})* is the position of construction object i and *(X_{Bl} , Y_{Bl})* is the coordinates of site boundary vertex *l*. The shortest distance of construction object j form border vector l can be determined from the following cross product:

[6] $$d_{lj} = \frac{\overrightarrow{V_l} \times \overrightarrow{V_{lj}}}{|\overrightarrow{V_l}|}$$

where $|\overrightarrow{V_l}|$is the length of border line l. To ensure that all construction objects are located inside the site boundary, equation Eq. 4 should be satisfied for all combinations of border lines and construction objects:

[7] $$d_{lj} \geq R_j \text{ , } for\ l = 1, \ldots, L\ and\ j = 1, \ldots, M$$

When an object is located on the right side of a border vector, the cross product of vector V_l and vector V_{lj} will have a negative value (d_{jl} <0). This will automatically refuse the position to be an acceptable answer. Eq.7 ensures that all the construction objects are located on the left side of the border vectors with a minimum distance equal to their radius.

2.4 Non-Overlap Constraint

When two objects exist on the site at the same time, they can not occupy the same space on the site. However, objects that belong to different periods of time are allowed to reuse the same space. To avoid overlap for two objects that exist on the site at the same time, they should have a minimum distance of their radiuses from each other. This can be presented by the following equations:

[8] $$D_{ij} \geq R_i + R_j$$

where D_{ij} is the distance between object i and j, and R_i and R_j are the radiuses of the objects i and j. When two objects do not have overlap, their distance is more than the required minimum, and accordingly, Eq.8 is satisfied. However, this equation does not reflect dynamic layout planning. Next section explains how the non-overlap constraint should be modified to reflect the duration of objects on the site.

2.5 Dynamic non-overlap Constraint

A dynamic layout model allows objects from different periods of time to reuse the same space on the site. For instance in Figure 1, the space of geotechnical lab (object A) can be used by rebar workshop, batch plant, or gravel depot (objects B, C, or E, respectively). However, if the non-overlap constraints presented in Eq. 8 is applied to all objects, none of them will be able to reuse the same space. A binary time index (T_{ij}) between all pairs of objects is defined in the developed model to indicate which pair of objects exist on the site at the same time and which pairs do not. This index is defined as follow:

[9] $$T_{ij} = \begin{cases} 1 & objects\ i\ and\ j\ exist\ at\ the\ same\ time\ and\ i \neq j \\ 0 & otherwise \end{cases}$$

The above time indices are applied to Eq. 8 to develop the dynamic non-overlap constraint:

[10] $$D_{ij} \geq (R_i + R_j)T_{ij}$$

In this equations when the two objects do not exist on the site at the same time (time index=0), the right side of the equation equals zero, and accordingly, the constraint will be satisfied regardless of the distance between objects. This means that the objects can occupy the same space on the site. This constraint enables the model to consider the effect of time on the optimization process.

2.6 Minimum Distance Constraint

In some cases two objects are required to have a minimum distance from one another. For example, a resting facility needs to have a minimum safety distance from a tower crane. This constraint can be satisfied by adding the minimum required distance (D_m) to the right side of equation Eq. 10:

[11] $$D_{ij} \geq (R_i + R_j + D_m)T_{ij}$$

2.7 Adjacency Constraint

Some objects are required to be located adjacent to each other. For instance a parking lot needs to be adjacent to the offices. This constraint is a special case for non-overlap constraint which can be reached by satisfying the following equation:

[12] $$D_{ij} = R_i + R_j$$

2.8 Solving Multi-Equations

To determine the optimum layout, the equations explained above should be solved using techniques from mathematics. Complicated problems such as dynamic layout planning can be solved using strong mathematic software tools such as GAMS and LINGO. These tools provide different solvers which each is useful for a specific type of mathematic problems. The developed model uses GAMS software and selects BARON solver for the layout optimization problem. This solver uses Branch-And-Reduce Optimization Navigator (BARON) technique which guarantees reaching the optimum answer for non-linear optimization problems (Sahinidis and Tawarmalani 2011). This solver can ensure reaching the optimum location of objects since the layout planning problem is modeled using linear and non linear equations (Eq. 1 to Eq. 12).

3 Computational Example

A numerical example is used from literature to demonstrate the capabilities of the developed model in generating dynamic site layouts and compare its results to other methods. This example was originally introduced by the authors to determine the optimum layout for a project with three site objects and six construction objects (Andayesh and Sadeghpour 2013). The site shape and the location of the site objects are presented in Figure 3. This figure can be used to determine the boundary vertices and border vectors which will be used to satisfy site boundary constraint. Table 1 presents the radiuses for minimum bounding circles and summarizes the durations that objects exist on the site in the total project duration of 10 months. Table 2 presents the closeness weights between the objects.

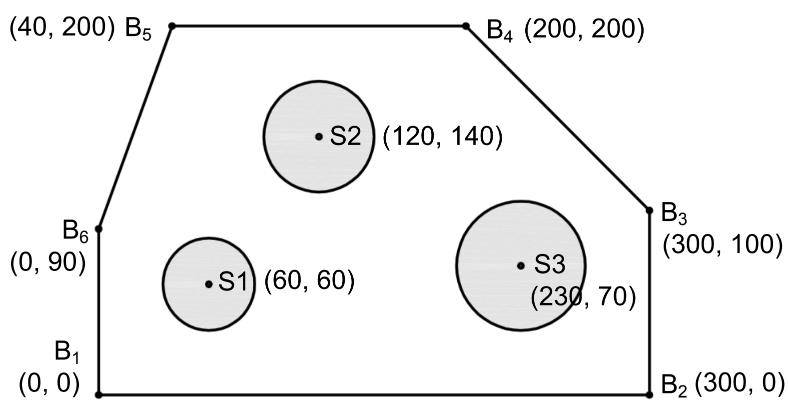

Figure 3: Site shape, boundary vertices, and location of site objects

The presented model was used to determine the optimum locations for the construction objects in the site. The time indices between objects were determined from the duration they exist on the site (Table 1). These binary indices, which are used to reflect the dynamic aspect of the model, are presented in Table 3. In this table index zero (0) shows that the two objects do not exist on the site at the same time, and accordingly, they can use the same space. On the other hand, index one (1) prevents the two objects from occupying the same space. Other mathematic equations were formed based on the information given in Tables 1 to 3 and Figure 1. The minimum utility function for the example was determined to be 40,454 which is exactly the same as the utility function found by Andayesh and Sadghpour 2013. Table 4 summarizes the optimum coordinates for the construction objects and Figure 4 presents the optimum layout graphically.

Table 1: Size and time of the existing objects

ID	Object Name	Size(m)	Time (months)
S1	Parking building	25	0–10
S2	Shopping mall	30	0–7
S3	Theatre hall	35	2–10
C1	Rebar workshop	15	0–7
C2	Material storage	17	3–9
C3	Batch plant	18	1–7
C4	Carpentry workshop	14	3–6
C5	Electrical tools storage	12	6–10
C6	Security office	15	7–10

Table 2: Closeness weights (workflows) between objects

ID	C1	C2	C3	C4	C5	C6
S1	120	-60	130	0	-70	150
S2	40	-80	40	90	50	100
S3	140	-140	135	-40	120	180
C1	-	0	0	0	0	0
C2	-	-	0	15	0	10
C3	-	-	-	0	0	0
C4	-	-	-	-	8	0
C5	-	-	-	-	-	20
C6	-	-	-	-	-	-

Table 3: Time indices between objects

ID	C1	C2	C3	C4	C5	C6
S1	1	1	1	1	1	1
S2	1	1	1	1	1	0
S3	1	1	1	1	1	1
C1	0	1	1	1	1	0
C2	1	0	1	1	1	1
C3	1	1	0	1	1	0
C4	1	1	1	0	0	0
C5	1	1	1	0	0	1
C6	0	1	0	0	1	0

Table 4: Optimum locations for the construction objects

ID	C1	C2	C3	C4	C5	C6
X	163.4	17.0	130.4	76.1	216.6	144.6
Y	76.7	17.0	76.8	142.4	115.1	90.0

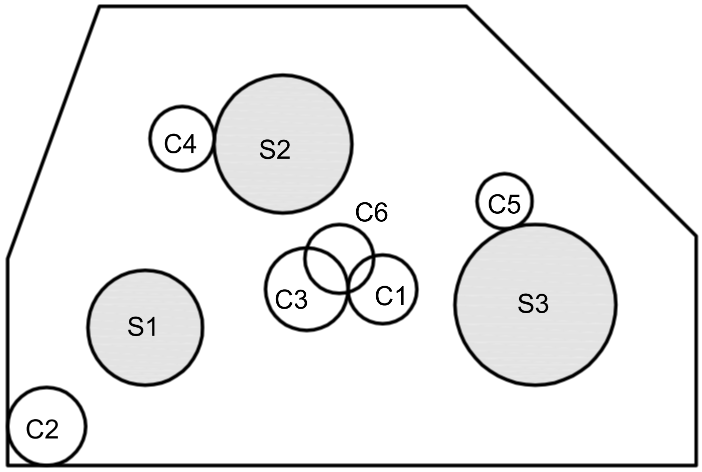

Figure 4: Optimum layout for the computational example

The optimum layout presented in Figure 4 shows that the security office (C6) has space overlap with the rebar workshop and the batch plant (C1 and C3). This is not a space conflict since the time index of C6 with C1 and C3 is zero (refer to Table 3) which means the security office does not exist on the site at the same time as the other two objects (see Table 1). In fact, when the rebar workshop (C1) and batch plant (C3) leave the site by the end of seventh (7th) month, the security office reuses their space during months eight to ten (8 to 10). On the other hand, the rebar workshop (C1) and batch plant (C3) are required for the same time (time index of 1 in Table 3), and accordingly, they become adjacent other than using the same space.

4 Concluding Remarks

This paper presented a dynamic layout planning model that is able to generate layouts that are optimized over the entire duration of the project. The model defines the objective function and planning constraints by several mathematical equations. This reflects the process of searching for the optimum layout to solving a mathematical optimization problem. The following planning constraints are addressed in this model: site boundary, non-overlap, dynamic planning, minimum distance, and adjacency. The site boundary constraint is satisfied using vector calculus to ensure that the construction objects are located inside the site. Non-overlap, minimum distance, and adjacency constraints are addressed by determining the distance between objects. Dynamic planning, which is the main concern for generating optimum layouts, is addressed by determining time indices for pair of objects. A time index of zero (0) is assigned to a pair of objects that do not exist on the site at the same time. This means that the overlap constraint is satisfied automatically and the objects can occupy the same space on the site. On the other hand objects with time index of one (1) exist on the site at the same time and can not use the same space. The generated multi equations then can be solved using the existing mathematical solvers such as GAMS software.

The main advantage of the mathematical model presented in this paper lies in its flexibility in addressing different constraints in layout planning. New constraints such as rectangular shapes for objects, rectilinear distances between them, and relocation costs can be added to the model to reflect the actual condition of a construction site. Furthermore, a mathematical based model can easily be connected to other planning software such as MS Project and AutoCAD to form a solid site layout planning tool.

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