



Simulation-based Optimization of Precast Box Girder Concrete Bridge Construction Using Full Span Launching Gantry

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Abstract: Several critical decisions have to be made by project planners during the planning and scheduling phases of construction projects. The numbers of crews and equipment are examples of such decisions. This paper presents a methodology for optimizing the construction of a precast box girder concrete bridge using full span launching gantry construction method. The limited experience with of similar projects increases the risk associated with using this construction method. Therefore, simulation of precast production, transportation, and placement cycles can be beneficial when planning new construction projects of this type. The optimization engine uses a fast messy Genetic Algorithm (fmGA) to minimize project cost and/or duration depending on the decision maker's requirements. A simulation model is used to estimate the project duration and cost for different scenarios representing combinations of resources that are generated by the fmGA. The interactions between the optimization and the simulation are explained. In addition, a probabilistic Pareto Front concept is proposed. The proposed model can be considered as a reliable tool for decision makers and planners. A hypothetical case study is presented to validate and demonstrate the applicability of the proposed approach.

1 Introduction

The highway infrastructures in North America are approaching, or have surpassed, their service life. Therefore, an intensive amount of reconstruction and rehabilitation work is expected on existing bridges. In order to expedite the construction process and reduce traffic interruption; transportation agencies may favor precast concrete bridges over cast in-situ concrete bridges. Several transportation agencies have shifted to Accelerated Bridge Construction (ABC) as an alternative to conventional construction methods. This recent shift was driven by the need to minimize traffic impacts which is caused by extended onsite construction activities. ABC refers to reducing the onsite construction time of bridges by using innovative planning, design, materials and construction methods (Federal Highway Administration, 2013). ABC has proven to have essential benefits over the conventional construction methods. These benefits can be noticed in the improved safety during construction, the higher quality and durability of the bridge, as well as in the reduction of onsite construction time, traffic impacts, social costs and environmental impacts. However, due to the unfamiliarity with a specific construction method, transportation agencies may choose more traditional construction methods that might not complete the project within the preferred time or budget limits.

Bridge construction operations require the use of a large number of construction equipment. In general, the goals behind selecting a fleet of equipment are: increase work safety, minimize cost, reduce equipment idle time, and maximize productivity. Operation cycles have many components, which vary in their durations, which make the analysis of productivity very difficult (Wright 1996, El-Moslmani 2002). Therefore, discrete event simulation can be used to measure the productivity of resources combinations and to analyze its efficiency. In their previous research, the authors have introduced a new approach based on discrete event simulation and 4D modeling for bridge reconstruction projects using full span launching gantry (Mawlana et al., 2012). Full span precast launching gantry method is done in two phases which are: (1) the casting of a full span length of a concrete box girder bridge at the casting yard;

and (2) the erection of the full span using a launching gantry onsite. The span length is typically between 30 m to 55 m and weighs between 600 tons to 1500 tons (NRS Bridge Construction Equipment , 2008). At the first phase, the full span box girder is cast, cured, pre-stressed and stored until the erection day. On the day of erection (phase 2), the precast girders are transported to the construction site by trailers. Then, the girders are loaded on to a trolley which travels along the completed section of the bridge to reach to the location of the span that will be launched as shown in Figure 1 (a). Next, the launching girder picks up the precast girder (Figure 1 (b)) and places it on the pier caps (Rajagopalan, 2006, Rosignoli, 2010). Finally, the launching gantry repositions to the new launching location as shown in Figure 1 (c). This method has been used before to achieve high quality bridges, reduce construction time and cost (Pan et al., 2008). Launching gantry is used to span over obstacles such as roads, expedite construction, reduce the need for scaffolding, and minimize the disruption to construction site (Benaim, 2008). However, this method requires high level of technology and has high equipment cost (Hewson, 2003).

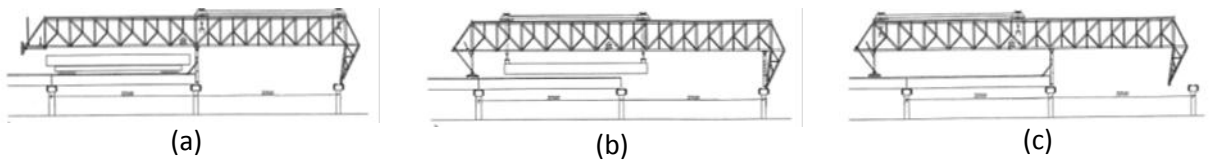


Figure 1: Operation of a Launching Gantry (Benaim, 2008)

The previously proposed approach (Mawlana et al., 2012) does not take into consideration the details of the construction method, the number of crews utilized, and the production of the concrete box girders. Therefore, a new framework is proposed for selecting the near optimal construction scenario considering the construction method, and the number of resources that minimizes the project cost, duration or both, and for facilitating the planning process (Mawlana and Hammad, 2013). The proposed simulation-based optimization framework can be used by project planners to enhance and improve the current practice of decision making in bridge construction operations. The construction scenario in this context consists of two main elements. The first element is the construction method that is used to construct a bridge. The second element is the decision variables related to that construction method. These decision variables are different from one construction method to another. The current paper aims to use this framework for the specific case of full span launching gantry construction method for the following objectives: (a) to find a near optimal project setting for constructing a bridge; (b) to evaluate the probabilities of near optimal solutions; and (c) to demonstrate the feasibility of the proposed approach using a case study.

2 Proposed Model

The proposed simulation-based optimization model (Figure 2) can be used to aid project planners in making decisions for bridges constructed using full span launching gantry method. The integration of discrete event simulation and fast messy Genetic Algorithm (fmGA) is shown in Figure 3. The outer loop (era) of the fmGA starts where each era (k) consists of the three phases of the Inner Loop of the fmGA. The process terminates when the current era (k) is larger than or equal to the maximum number of eras (k_{max}). This integration is described in more detail in (Mawlana and Hammad, 2013). The aim of the model is to select a near-optimal project setting using this construction method that satisfies a set of predefined objectives. A project setting refers to the decision variables planners need to make, such as the number of stressing crew, the number of equipment, the overtime policy, and the location of casting yard. Darwin optimization framework (Wu et al., 2012), which utilizes an fmGA, is used to solve the optimization problem. The objective function of this model can be a single objective (i.e. cost or duration), or multi-objective (i.e. cost and duration). The model takes into account fourteen decision variables which are: (1) the number of delivery trucks, (2) the number of onsite trolleys, (3) the distance of the casting yard from the access of the construction site, (4) the number of rebar cage molds, (5) the number of inner molds, (6) the number of outer molds, (7) the number of preparation crews, (8) the number of stressing crews, (9) the number of steel crews, (10) the number of casting crews, (11) the curing method, (12) the overtime

policy, (13) the casting yard storage capacity, and (14) the storage time of the span in the casting yard. Each of these decision variables has an impact on the project duration and cost. The overtime policy used for this model is obtained from (Orabi et al., 2009, RSMean Engineering Department, 2011) and it is summarized in Table 1. Six policies are considered in this model. Each policy has a different number of working hours per day and the number of working days per week. The impact of working overtime on productivity is based on the average loss of productivity over a four week period. The cost adjustment factor represents the increase in cost due to working overtime based on double times the regular wage.

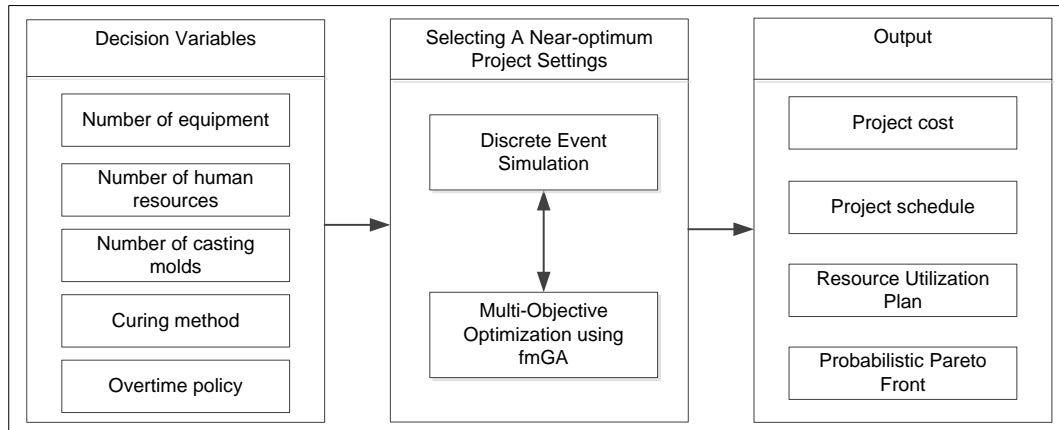


Figure 2: Simulation-based Optimization Model

Table 1: Overtime Policy

Policy	Working Calendar	Shifts/Day	Productivity Adjustment Factor (%)	Cost Adjustment Factor (%)
1	8 hours/5days	1	100.00	100.00
2	12 hours/5days	1	76.25	133.30
3	24 hours/5days	2	68.75	153.30
4	8 hours/7days	1	88.75	128.60
5	12 hours/7days	1	68.75	152.40
6	24 hours/7days	2	62.00	175.25

2.1 Probabilistic Pareto Front

In general, the multi-objective functions of the genetic algorithm should return the same values for the same set of decision variables. This is only true when a closed-form formula or deterministic simulation is used. However, when stochastic simulation is used, a different objective value is obtained for the same set of decision variables. This is due to the fact that the durations of tasks within the stochastic simulation model are modeled by distribution functions. Therefore, each time the simulation is executed a random seed is used to determine the duration of a task. In order to overcome this problem, a fixed seed is used to execute the simulation model. By doing this, the same objective values are obtained from the same set of decision variables used in the simulation model. In addition, the occurrence probability of each solution is calculated to represent the confidence level of that solution. This is done by using the sets of decision variables representing the Pareto solutions. For each set of decision variables, simulation is run a large number of times in order to obtain a distribution function. Using the values of the objective functions obtained from the Pareto front and the distribution function for each set of decision variables, the probability of occurrence of each Pareto front solution is calculated. This method can help the decision makers to select the near-optimum solution that meets their objectives.

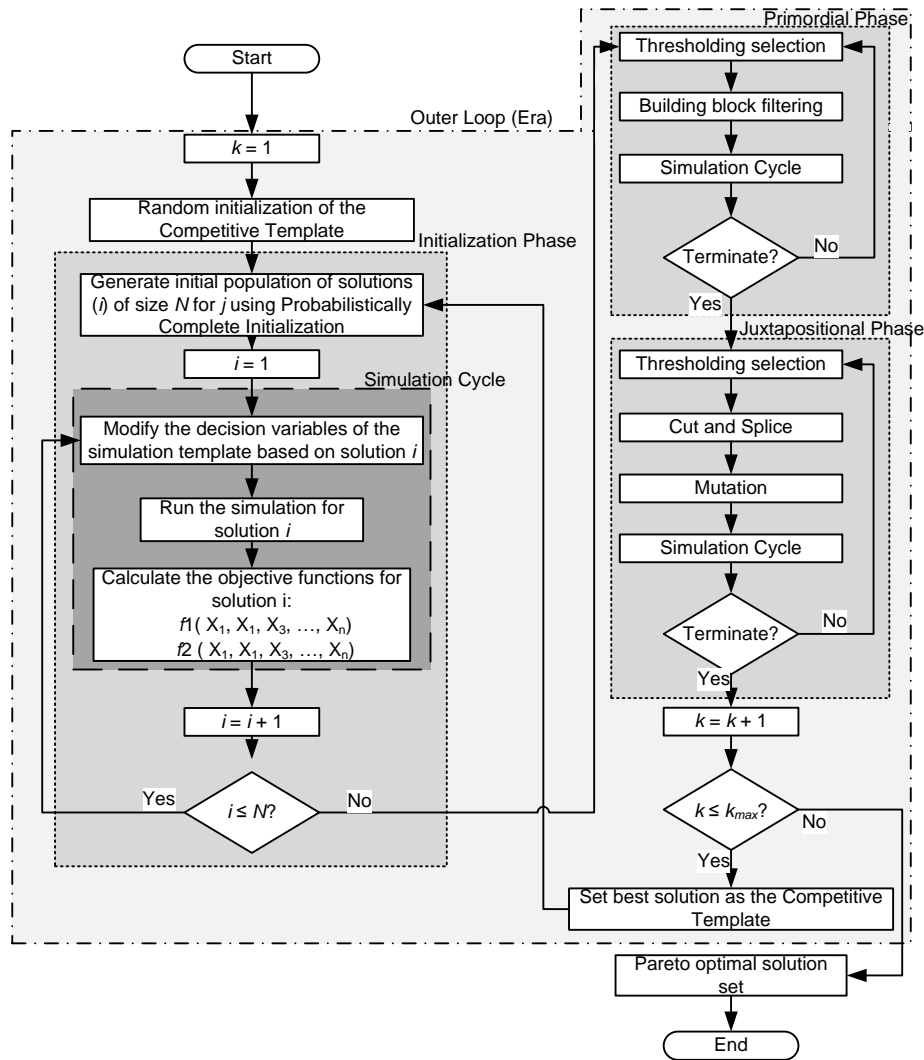


Figure 3: Integration between Discrete Event Simulation and Fast Messy Genetic Algorithm

2.2 Construction Operations Simulation Models

The simulation models are implemented in STROBOSCOPE, an acronym for State- and Resource-Based Simulation of CONstruction ProcEsses, which was designed for simulating construction operations (Martínez, 1996). STROBOSCOPE can accommodate deterministic and stochastic modeling of construction operations. This allows for extensive sensitivity analysis to be carried out which in turn reduces the risk associated with the project. The developed simulation model of bridge construction using full span launching gantry is shown in Figure 4. This construction method has been used in several projects around the world (Hewson, 2003, Rajagopalan, 2006, Benaim, 2008, Rosignoli, 2010). The labels of the arrow represent the flow of the resources in the model. For example, ST1 indicates that the *Steel_Crew* is flowing from the queue and to the *BottomSlab_Web* task. The simulation starts by initializing the queues that hold the resources needed for the construction operations. The steel crew starts placing the steel reinforcement and the tendons' ducts for the bottom slab and the webs of the full precast span using a rebar mold. Then, an inner mold is loaded to the finished rebar cage. Afterwards, the steel crew places the steel reinforcement for the top slab. Next, the finished rebar cage is placed in an outer mold. Then, the casting crew casts the span. At this point, the casted span undergoes the curing process. Afterwards, the inner mold is removed and the first stage of post-tensioning is performed by the

operation from the casting operation to the launching operation. Equations [6], and [7] are used to estimate the total number of working days and the total project duration in calendar days; respectively.

$$[1] \text{ Total Project cost} = \text{Mobilization cost} + \text{Indirect cost} + \text{Direct cost}$$

$$[2] \text{ Mobilization Cost} = 2 \times \sum_{i=1}^n E_i \times N_i + \sum_{k=1}^m C_k \times M_k$$

$$[3] \text{ Indirect Cost} = \text{Total project duration} \times \text{Daily indirect cost}$$

$$[4] \text{ Direct Cost of Equipment} = \sum_{i=1}^n RE_i \times E_i \times TE_i$$

$$[5] \text{ Direct Cost of Crews} = \sum_{k=1}^m RC_k \times C_k \times TC_k \times \text{Cost adjustment factor}$$

$$[6] \text{ Total Working Days} = \frac{\text{Total simulation time}}{\text{Working hours per day} \times 60 \left(\frac{\text{min}}{\text{hr}}\right)}$$

$$[7] \text{ Total Project Duration} = \text{Total working days} + \left\lfloor \frac{\text{Total working days}}{\text{Working days per week}} \right\rfloor \times \left[7 \left(\frac{\text{days}}{\text{week}}\right) - \text{Working days per week} \right]$$

where n is the number of equipment types utilized in the project; E_i is the number of equipment of type i utilized in the project; N_i is the mobilization cost of an equipment of type i ; m is the number of crew types utilized in the project; C_k is the number of crews of type k utilized in the project; M_k is the mobilization cost of a crew of type k ; RE_i is the hourly cost of an equipment of type i ; TE_i is the number of hours equipment of type i is assigned to the project; RC_k is the hourly cost of a crew of type k ; TC_k is the number of hours crew of type k is assigned to the project; and $\lfloor x \rfloor$ is $\max \{m \in \mathbb{Z} \mid m \leq x\}$.

3 Case Study

A hypothetical case study is presented here to demonstrate the applicability of the proposed model. The case study consists of constructing a precast concrete box girder bridge using full span launching gantry method. The bridge consists of 500 spans with identical spans measuring 25 m. Table 2 shows the durations of the tasks used in this study. The durations of the casting operation tasks were adapted from (Marzouk et al., 2007), and it is assumed that the tasks' duration are linearly related to the length of the span. The durations of the launching process tasks are adapted from (VSL International Ltd., 2013) by adding a range of $\pm 25\%$ in order to have a distribution for the durations. Most of the tasks' durations are represented by a distribution to model the uncertainty associated with this construction method. It is assumed that these durations are based on a single crew which consists of 4 workers for estimating the cost of the project. The *Span_Curing* task has duration of 1200 or 600 minutes depending on the curing method used. In this case study, it is assumed that there are two curing methods namely regular and accelerated. Traveling tasks, such as *Trailer Haul*, are represented as functions of distance and speed. Table 3 summarizes the decision variables considered in the optimization to be optimized along with their minimum, maximum, and increment value. Table 4 shows the fmGA configuration used in this study. The fmGA uses two operators which are the cut and splice operator, and the mutation operator. The purpose of the cut and splice operators is similar to the crossover operator used in simple Genetic Algorithms which is to create new strings by combining genes from different strings. In fmGA, this operation is done in two steps. At first, a string is cut into two parts based on the specified cut rate. Then, the splice operator recombines two strings based on the specified splice rate (Goldberg et al., 1993). This optimization is run for 1,000,000 trials through 20 eras where each era consists of 500 generations and each generation has a population of 100 strings. This model was run on an Intel Core i7, Quad-core processor, 3.4 GHz machine with 4 GB RAM and it took 61 hours to finish the optimization.

Table 2: Tasks Durations Used in the Simulation Model

Task	Duration (minutes)	Task	Duration (minutes)
BottomSlab_Web	Normal [1673, 165.84] *	Trailer_Loading	Triangular[45, 60, 75] **
Inner_Mold	Uniform[120, 480] *	Trailer Haul	F (Distance, Speed)
TopSlab	Normal[1979, 281.69] *	Trolley_Loading	Triangular[45, 60, 75] **
LiftToMold	Triangular[30, 45, 60]	Trailer_Return	F (Distance, Speed)
Cast_Span	Normal[1544, 75.24] *	Trolley_Travel	F (Distance, Speed)
Span_Curing	(1200 or 600) *	Reposition	Triangular[180, 240, 300]
RemoveInnerMol	Uniform [90,240] *	Erection_Span	Triangular[180, 240, 300] **
Posttension_1st	Uniform[120,360] *	Trolley_Return	F (Distance, Speed)
LiftToStorage	Triangular[45, 60, 75] **	Prepare_Bearing	Triangular[180, 240, 300] **
Posttension_2nd	Uniform[120,360] *	Load_Transfer	Triangular[45,60, 75] **

* Adapted from (Marzouk et al., 2007)

** Adapted taken from (VSL International Ltd. , 2013)

Table 3: Decisions Variables Used in the Optimization

Index in Solution Representation	Decision Variable	Minimum	Maximum	Increment
1	Number of delivery trucks	1	20	1
2	Number of onsite trolleys	1	4	1
3	Precast yard distance (km)	10	100	10
4	Number of rebar cage molds	1	10	1
5	Number of inner molds	1	10	1
6	Number of outer molds	1	10	1
7	Number of preparation crews	1	4	1
8	Number of stressing crews	1	4	1
9	Number of steel crews	1	4	1
10	Number of casting crews	1	4	1
11	Precast yard storage capacity	1	50	5
12	Storage time (hr)	1	84	1

The model was used to optimize the hypothetical bridge construction and was able to generate a set of solutions where each solution represents a project setting. As shown in Figure 5, the Pareto front solutions provide non-dominated tradeoff between minimizing the project duration and minimizing the construction cost. A tradeoff exists because reducing the project duration requires the use of extra resources which in turn will increase the project cost. The gap between solutions is caused by the overtime policy used. It can be noticed that only overtime policies (OP) 1, 4, and 5 have been selected by the optimization algorithm. The impact of accelerating the project by working multiple shifts on the project cost can be noticed by comparing *Solution 1* and *Solution 21* as shown in Table 5. The solution representation shows the number of resources, and the settings of the casting yard. Each decision variable has an index in the solution representation array as shown in Table 3. For example, the third

element of the array represents the distance from the precast yard to the access point of the construction site. For example, in *Solution 4*, the distance is 100 km for and this solution requires the use of accelerated curing method and overtime policy 5 which is 12 hours per day and 7 days per week as shown in Table 1. *Solution 1* requires 921 days and \$4,464 million to finish the construction compared to 1,344 days and \$1,829 million as required by *Solution 21*. *Solution 1* reduces the project duration by almost half compared to *Solution 21* but it will cost almost 2.5 times more. Between these two extremes, there are other feasible solutions from which the decision maker can select. For example, *Solution 15* requires 1,084 days and \$2,418 million to finish the project. *Solution 1* represents the shortest project duration and the most expensive project alternative. It also represents the highest transportation agency expenditure and the shortest duration of public inconvenience or traffic interruption. On the other extreme, *Solution 21* represents the longest project duration and the cheapest alternative.

Table 4: fmGA Configuration Used in the Case Study

Parameter	Value
Cut rate	0.017
Splice rate	0.6
Mutation rate	0.015
Random seed	0.5
Population size	100
Generations per era	500
Number of eras	20
Maximum trials	1,000,000
Size of search space	$2,064,384 \times 10^6$

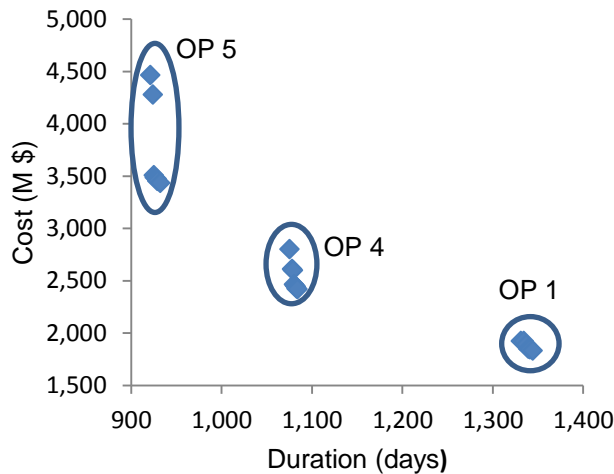


Figure 5: Pareto Front with the Tradeoff between Project Cost and Duration

To demonstrate the feasibility of the probabilistic Pareto Front, *Solution 13* and *Solution 15* are compared. *Solution 13* has a probability of 0.74% to finish the project within 1,082 days while *Solution 15* has a probability of 21.40% to finish the project within 1,084 days. Traditionally, decision makers will compare the two solutions and find out that *Solution 13* reduces project duration by almost 0.18% while *Solution 2* reduces cost by 0.95%. Based on this comparison, the difference between the two solutions is very small. However, using the probability of the solutions, it can be noticed that the likelihood of *Solution 15* finishing with 1,084 days is almost 45 times the likelihood of *Solution 13* finishing within 1,082 days.

This case study emphasizes the capabilities and usefulness of the developed model in generating different alternatives of project settings. Decision makers and project planners can evaluate the Pareto front solutions and select the alternative that best meet their requirements.

Table 5: Details of the Pareto Front Set of Solutions

Solution	Duration (days)	Cost (M \$)	Probability (%)	Solution Representation	Curing Method	Overtime Policy
1	921	4,464	0.002	{1,3,,20,7,10,10,4,1,4,2,16,2}	Regular	5
2	924	4,279	0.01	{2,1,40,10,7,10,1,2,4,4,36,6}	Accelerated	5
3	925	3,507	0.06	{1,1,100,6,9,10,1,1,4,2,21,9}	Accelerated	5
4	926	3,484	0.05	{1,1,100,10,10,8,1,1,4,2,16,9}	Accelerated	5
5	928	3,472	0.13	{1,1,60,10,10,6,1,1,4,2,16,22}	Accelerated	5
6	929	3,451	0.19	{1,1,20,6,10,5,1,1,4,2,16,42}	Accelerated	5
7	932	3,435	10.57	{1,1,100,7,10,4,1,1,4,2,11,10}	Accelerated	5
8	1,075	2,801	0.06	{2,2,100,9,10,10,1,1,4,2,50,3}	Accelerated	4
9	1,078	2,612	0.42	{2,1,10,8,10,9,1,2,4,2,16,15}	Accelerated	4
10	1,079	2,599	0.58	{2,1,100,7,9,7,2,1,4,2,16,25}	Accelerated	4
11	1,080	2,465	0.78	{1,1,20,10,9,5,1,1,4,2,26,11}	Accelerated	4
12	1,081	2,450	0.47	{2,1,100,9,8,7,1,1,4,2,11,18}	Accelerated	4
13	1,082	2,441	0.74	{1,1,100,10,9,10,1,1,4,2,16,27}	Accelerated	4
14	1,083	2,424	12.29	{1,1,70,10,9,10,1,1,4,2,11,3}	Accelerated	4
15	1,084	2,418	21.40	{1,1,10,9,8,10,1,1,4,2,11,2}	Accelerated	4
16	1,331	1,924	0.002	{1,1,40,9,9,10,1,1,4,2,31,6}	Accelerated	1
17	1,334	1,923	0.01	{2,1,20,7,9,10,1,1,4,2,21,7}	Regular	1
18	1,337	1,888	0.18	{2,1,40,9,10,4,1,1,4,2,16,2}	Accelerated	1
19	1,339	1,850	0.28	{1,1,20,10,9,5,1,1,4,2,16,3}	Accelerated	1
20	1,341	1,842	2.14	{1,1,90,10,10,6,1,1,4,2,11,1}	Accelerated	1
21	1,344	1,829	1.67	{1,1,20,10,9,4,1,1,4,2,11,11}	Accelerated	1

4 Summary and Future Work

A model for optimizing the construction of a precast box girder concrete bridge using full span launching gantry construction method is presented in this paper. This model will aid project planners and decision makers to select the near optimum project settings for constructing bridges using this method. Simulation models of the precast production, the transportation, and the placement cycles were developed to estimate the cost and duration of new construction projects using this construction method. Simulation was coupled with fast messy Genetic Algorithm (fmGA) to minimize the project cost and duration. In addition, the proposed probabilistic Pareto Front will aid the decision makers in determining the confidence of the nominated solutions. The proposed model can be considered as a reliable tool for decision making. A case study was presented to validate and demonstrate the applicability of the proposed approach.

Future work of this research will include: (1) conducting a survey to collect real data regarding the duration of construction tasks related to the full span launching gantry construction method; (2) validating

the developed simulation model; and (3) examining the impact of different fmGA configurations and simulation seed on finding the near-optimum solution.

Acknowledgment

The authors would like to acknowledge the support of Bentley Inc. for providing the Darwin Optimization Framework. Furthermore, the help and support provided by Dr. Zheng Yi Wu is highly appreciated.

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